On the Polarization Diversity of Aperture Array Antennas for SKA Wide-Field Polarimetry

Rob Maaskant and Tobia Carozzi

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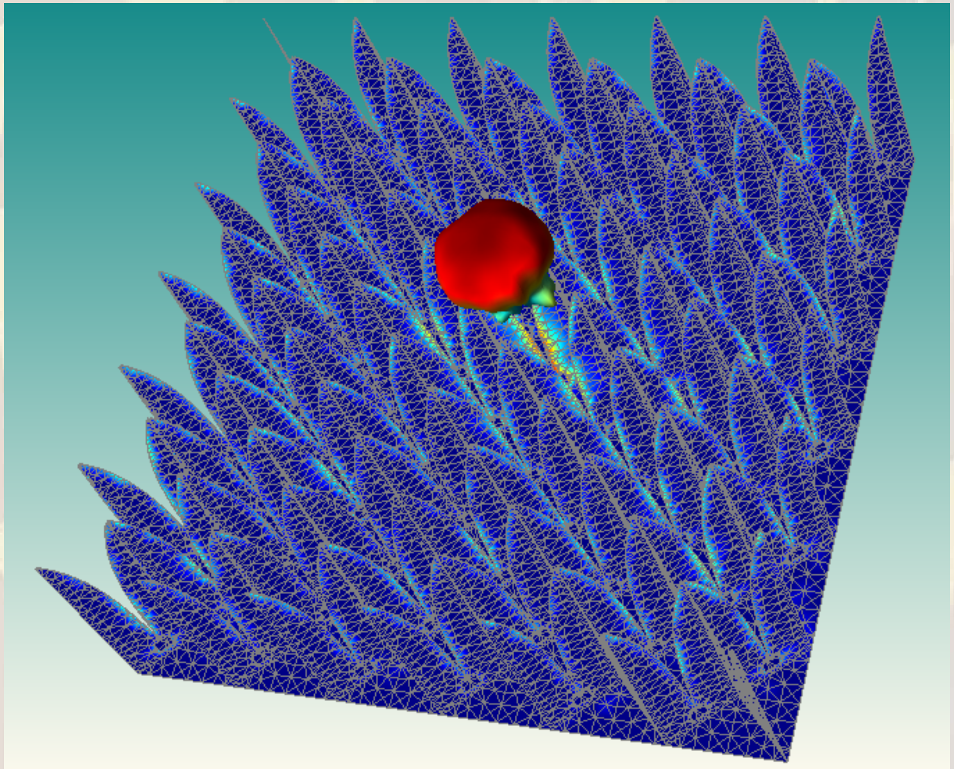
BYU workshop, Provo, Utah



In part based on a discussion with:
Karl Warnick, Brian Jeffs, Marianna Ivashina, and Stefan Wijnholds

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Specifications on Polarization Purity for the Square Kilometre Array

SKA MEMO 100, “preliminary specifications for the SKA” (2007, page 24)

Pulsar timing experiments
Calibrated polarization purity has to be at the level of –40 dB at the center of the field.

Epoch of Reionisation (EoR) and Cosmic Magnetism experiments
A purity of at least –30 dB over the entire field is required (leakage of intensity signal into the other Stokes parameters, after calibration and mosaicing).

“Experiments will be needed to verify the level of polarization purity that can be achieved at the output of polarization-optimized beam formers.”

- Polarization purity can be improved to virtually any level by compromising the SNR?
- Polarization purity should be a positive number?
- George Heald: “ max. 0.5% (-23 dB) of the total intensity is allowed to leak into the other Stokes parameters (over the entire FoV)”
- How are these numbers related to standard IEEE definitions on cross-polarization?

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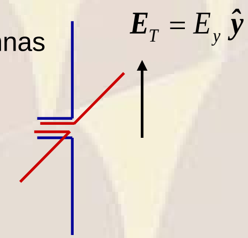
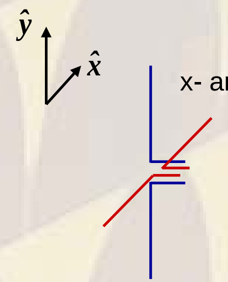
IEEE Standard Definitions on Cross-Polarization

- **Cross-Polarization Discrimination (XPD):** The ratio of the power level at the output of a receiving antenna, nominally co-polarized with the transmitting antenna, to the output of a receiving antenna of the same gain but nominally orthogonally polarized to the transmitting antenna

x- and y-oriented transmitting antennas



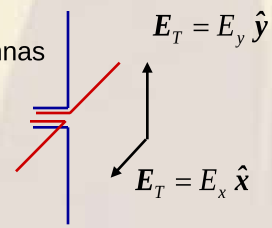


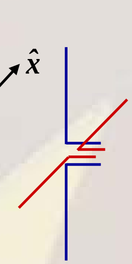
x- and y-oriented receiving antennas

$$\mathrm{XPD_y} = \frac{P_y^{\mathrm{out}}}{P_x^{\mathrm{out}}}$$

- **Cross-Polarization Isolation (XPI):** The ratio of the wanted power to the unwanted power in the same receiver channel when the transmitting antenna is radiating nominally orthogonally polarized signals at the same frequency and power level

x- and y-oriented transmitting antennas



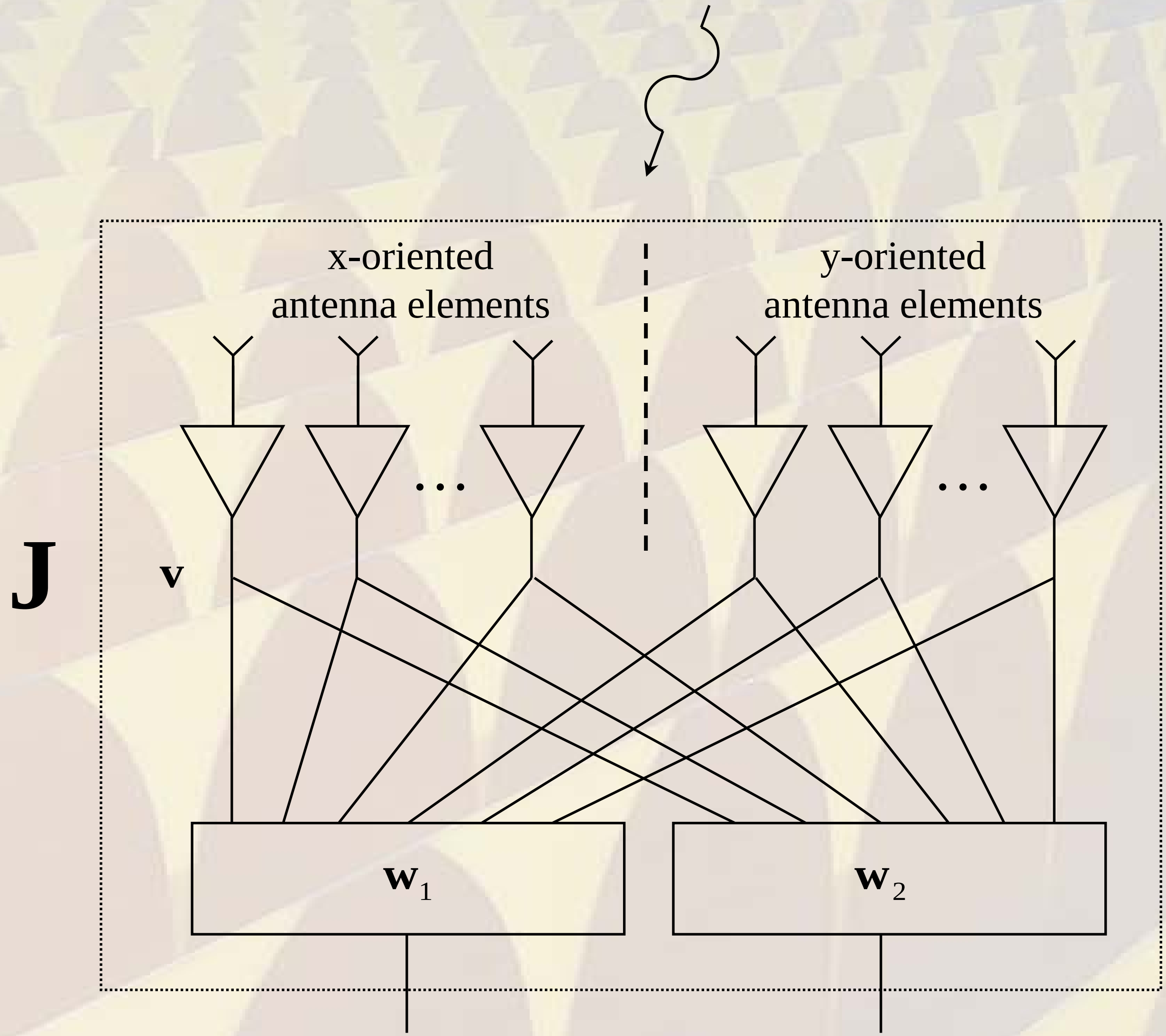


x- and y-oriented receiving antennas

$$\mathrm{XPI_y} = \frac{\left. P_y^{\mathrm{out}} \right|_{\boldsymbol{E}_T = E_y\hat{\boldsymbol{y}}}}{\left. P_y^{\mathrm{out}} \right|_{\boldsymbol{E}_T = E_x\hat{\boldsymbol{x}}}}$$

A Radio Polarimeter

$$\boldsymbol{E}^{\text{inc}} = E_{\text{u}}\hat{\mathbf{u}} + E_{\text{v}}\hat{\mathbf{v}}$$



$$v_1^{\text{out}} = \mathbf{w}_1^H \mathbf{v} \qquad v_2^{\text{out}} = \mathbf{w}_2^H \mathbf{v}$$

$$\begin{bmatrix} v_1^{\text{out}} \\ v_2^{\text{out}} \end{bmatrix} = \underbrace{\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}}_{\mathbf{J}} \begin{bmatrix} E_{\text{u}} \\ E_{\text{v}} \end{bmatrix}$$

Jones matrix
(Hamaker, Bregman, Sault)

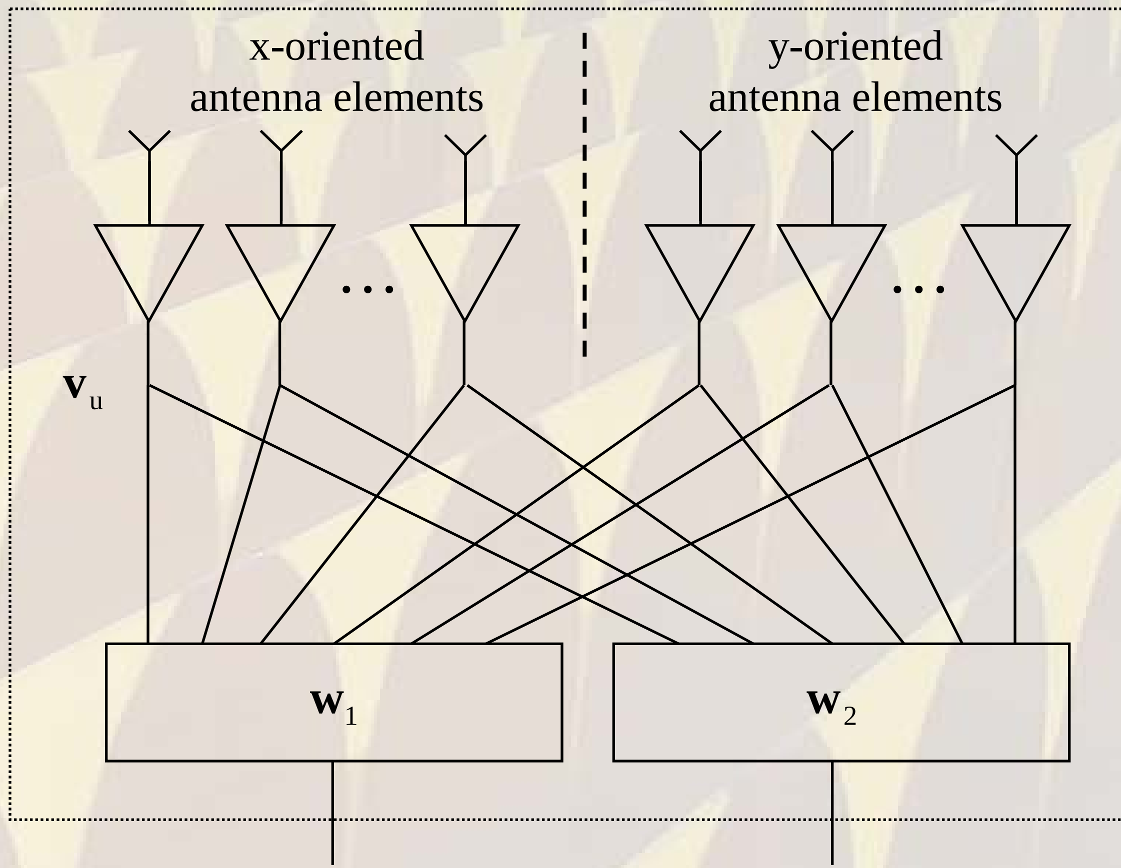
Characterization of the Radio Polarimeter – Determination of the Jones matrix

$$\mathrm{XPD}_u = \frac{|J_{11}|^2}{|J_{21}|^2}$$

$$\left(E_\mathrm{u} = 1,\ E_\mathrm{v} = 0\right)$$

$$\boldsymbol{E}^{\mathrm{inc}} = \hat{\mathbf{u}}$$

$$\begin{bmatrix} v_1^{\mathrm{out}} \\ v_2^{\mathrm{out}} \end{bmatrix} = \mathbf{J} \begin{bmatrix} E_\mathrm{u} \\ E_\mathrm{v} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} J_{11} \\ J_{21} \end{bmatrix}$$



$$J_{11} = \mathbf{w}_1^H \mathbf{v}_\mathrm{u} \qquad J_{21} = \mathbf{w}_2^H \mathbf{v}_\mathrm{u}$$

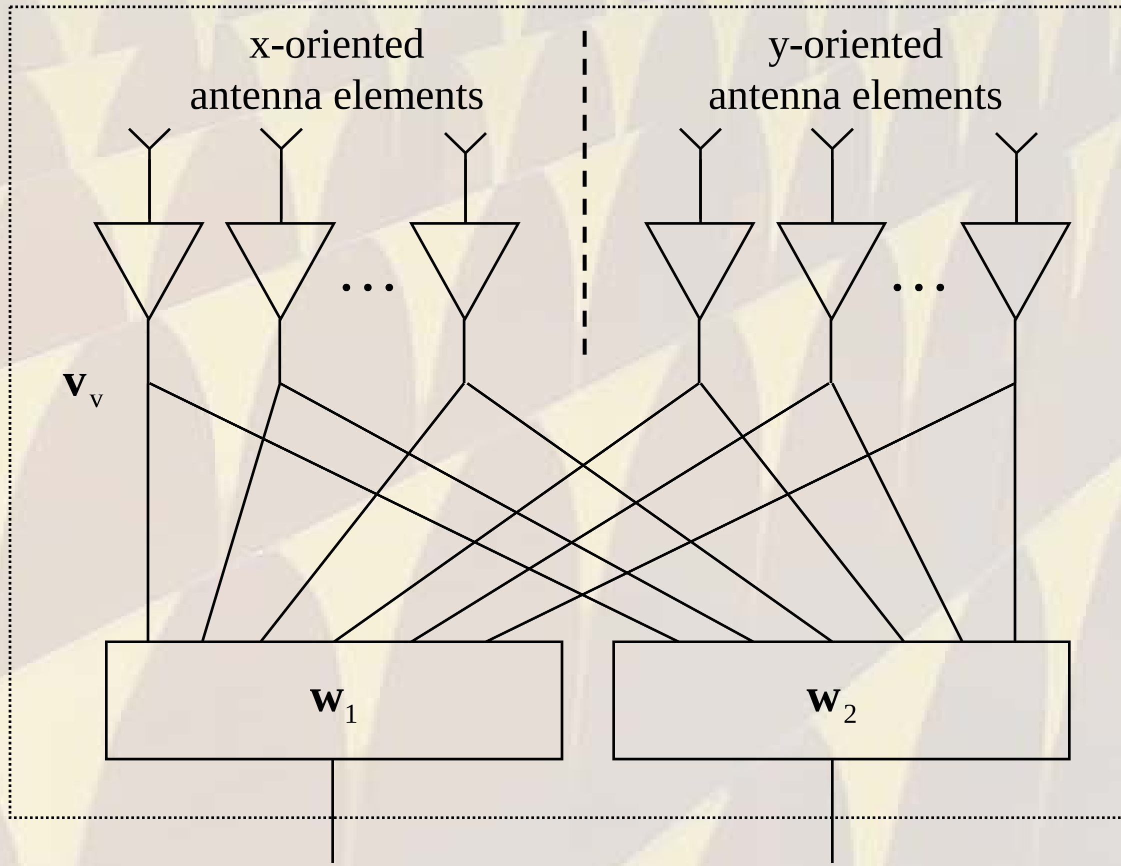
Characterization of the Radio Polarimeter – Determination of the Jones matrix

$$\mathrm{XPD}_u = \frac{|J_{11}|^2}{|J_{21}|^2}$$

$$\mathrm{XPD}_v = \frac{|J_{22}|^2}{|J_{12}|^2}$$

$$\left(E_\mathrm{u} = 0, E_\mathrm{v} = 1\right)$$

$$\boldsymbol{E}^{\mathrm{inc}} = \hat{\boldsymbol{v}}$$



$$J_{12} = \mathbf{w}_1^H \mathbf{v}_\mathrm{v} \qquad J_{22} = \mathbf{w}_2^H \mathbf{v}_\mathrm{v}$$

$$\begin{bmatrix} v_1^{\mathrm{out}} \\ v_2^{\mathrm{out}} \end{bmatrix} = \mathbf{J} \begin{bmatrix} E_\mathrm{u} \\ E_\mathrm{v} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} J_{12} \\ J_{22} \end{bmatrix}$$

$$\mathbf{J} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{w}_1^H \mathbf{v}_\mathrm{u} & \mathbf{w}_1^H \mathbf{v}_\mathrm{v} \\ \mathbf{w}_2^H \mathbf{v}_\mathrm{u} & \mathbf{w}_2^H \mathbf{v}_\mathrm{v} \end{bmatrix}$$

$$\mathrm{XPI}_1 = \frac{|J_{11}|^2}{|J_{12}|^2}$$

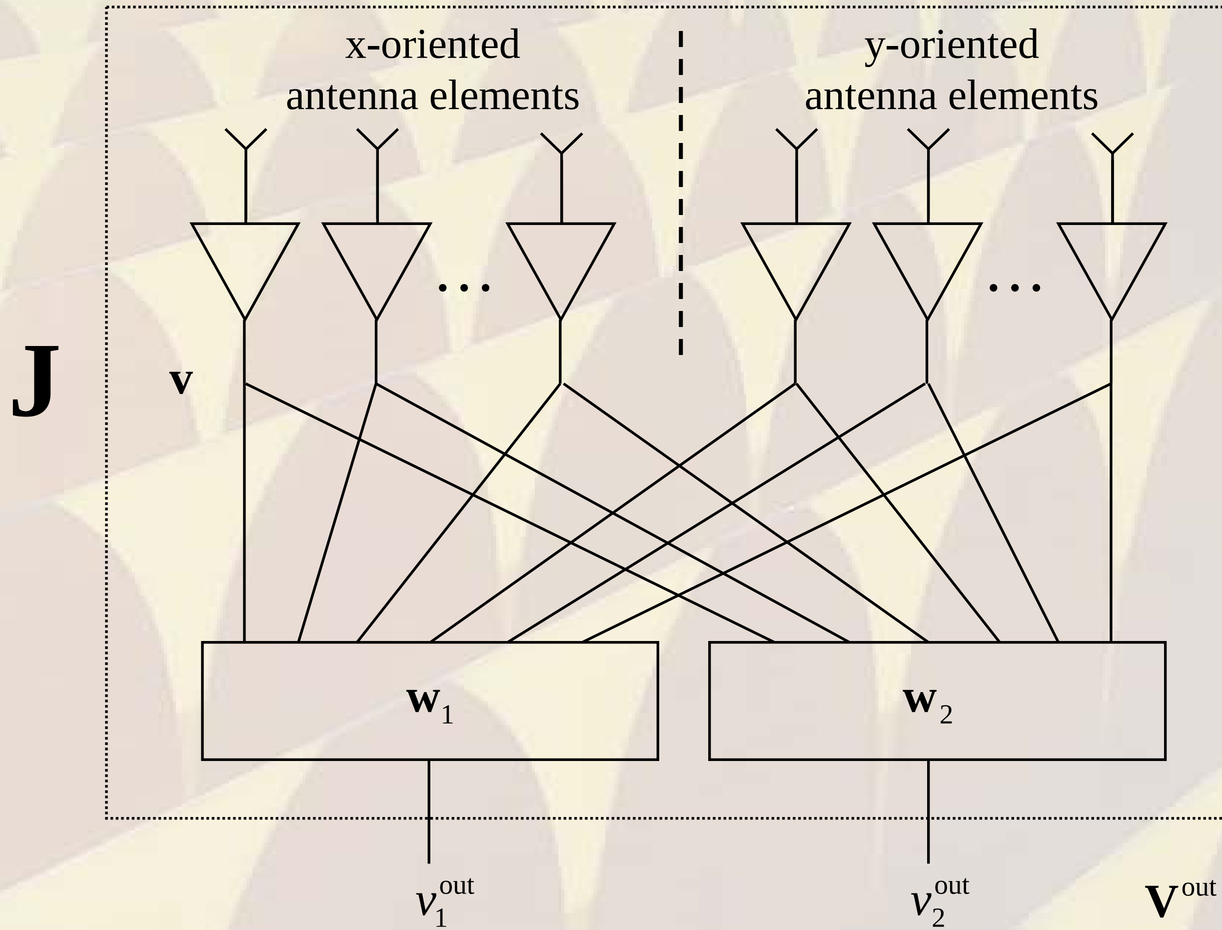
$$\mathrm{XPI}_2 = \frac{|J_{22}|^2}{|J_{21}|^2}$$

The Jones matrix, the XPDs and the XPIs depend on the weight vectors and the chosen reference frame

A Radio Polarimeter

$$\boldsymbol{E}^{\text{inc}} = E_{\text{u}}\hat{\mathbf{u}} + E_{\text{v}}\hat{\mathbf{v}}$$

$$\mathbf{R}_{\boldsymbol{E}} = \begin{bmatrix} \overline{\left|E_{\text{u}}\right|^2} & \overline{E_{\text{u}}\left(E_{\text{v}}\right)^*} \\ \overline{E_{\text{v}}\left(E_{\text{u}}\right)^*} & \overline{\left|E_{\text{v}}\right|^2} \end{bmatrix}$$



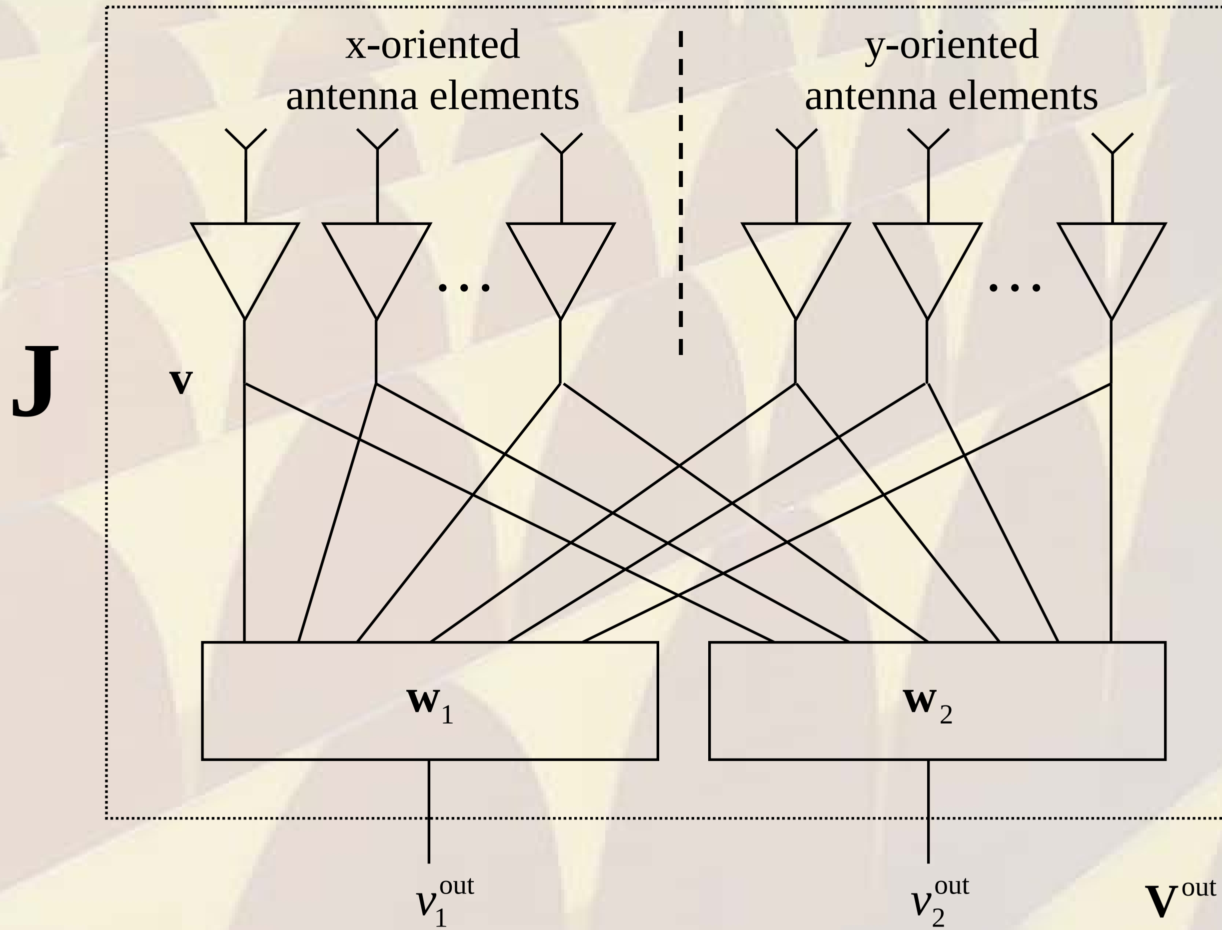
A Radio Polarimeter

$$\mathbf{V}^{\text{out}} = \begin{bmatrix} \overline{\left|v_1^{\text{out}}\right|^2} & \overline{v_1^{\text{out}}\left(v_2^{\text{out}}\right)^*} \\ \overline{v_2^{\text{out}}\left(v_1^{\text{out}}\right)^*} & \overline{\left|v_2^{\text{out}}\right|^2} \end{bmatrix}$$

A Radio Polarimeter

$$\boldsymbol{E}^{\text{inc}} = E_{\text{u}}\hat{\mathbf{u}} + E_{\text{v}}\hat{\mathbf{v}}$$

$$\mathbf{R}_{\boldsymbol{E}} = \begin{bmatrix} \overline{|E_{\text{u}}|^2} & \overline{E_{\text{u}}(E_{\text{v}})^*} \\ \overline{E_{\text{v}}(E_{\text{u}})^*} & \overline{|E_{\text{v}}|^2} \end{bmatrix}$$



Measurement equation (Hamaker, Bregman, Sault)

$$\mathbf{V}^{\text{out}} = \mathbf{J}\mathbf{R}_{\boldsymbol{E}}\mathbf{J}^H$$

Ideally: $\mathbf{J} = G\mathbf{I}$

$$\mathbf{V}^{\text{out}} = \begin{bmatrix} \overline{|v_1^{\text{out}}|^2} & \overline{v_1^{\text{out}}(v_2^{\text{out}})^*} \\ \overline{v_2^{\text{out}}(v_1^{\text{out}})^*} & \overline{|v_2^{\text{out}}|^2} \end{bmatrix}$$

How ideal is the Jones matrix ?

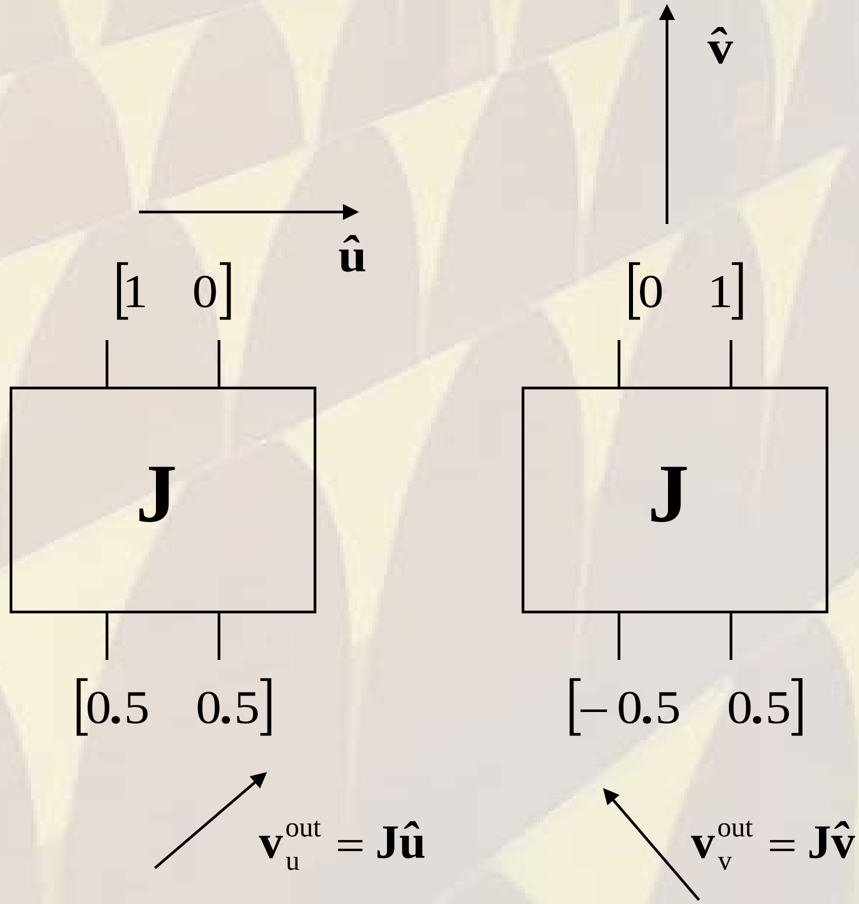
$$\begin{bmatrix} v_1^{\text{out}} \\ v_2^{\text{out}} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} E_{\text{u}} \\ E_{\text{v}} \end{bmatrix}$$

Condition number κ of $\mathbf{J}$ measures how close we achieved the situation that $\mathbf{J} = G\mathbf{I}$. Important when we aim to solve $\boldsymbol{E}^{\text{inc}} = \mathbf{J}^{-1}\mathbf{v}^{\text{out}}$ with high accuracy

The Jones polarimeter’s *intrinsic cross-polarization ratio* (Tobia Carozzi): $\text{IXR}_{\text{J}} = \left(\dfrac{\kappa(\mathbf{J})+1}{\kappa(\mathbf{J})-1} \right)^2$

$$\kappa(\mathbf{J}) \to 1, \quad \text{IXR}_{\text{J}} \to \infty$$
$$\kappa(\mathbf{J}) \to \infty, \quad \text{IXR}_{\text{J}} \to 1$$

Example: Jones matrix which rotates and scales the polarization vectors



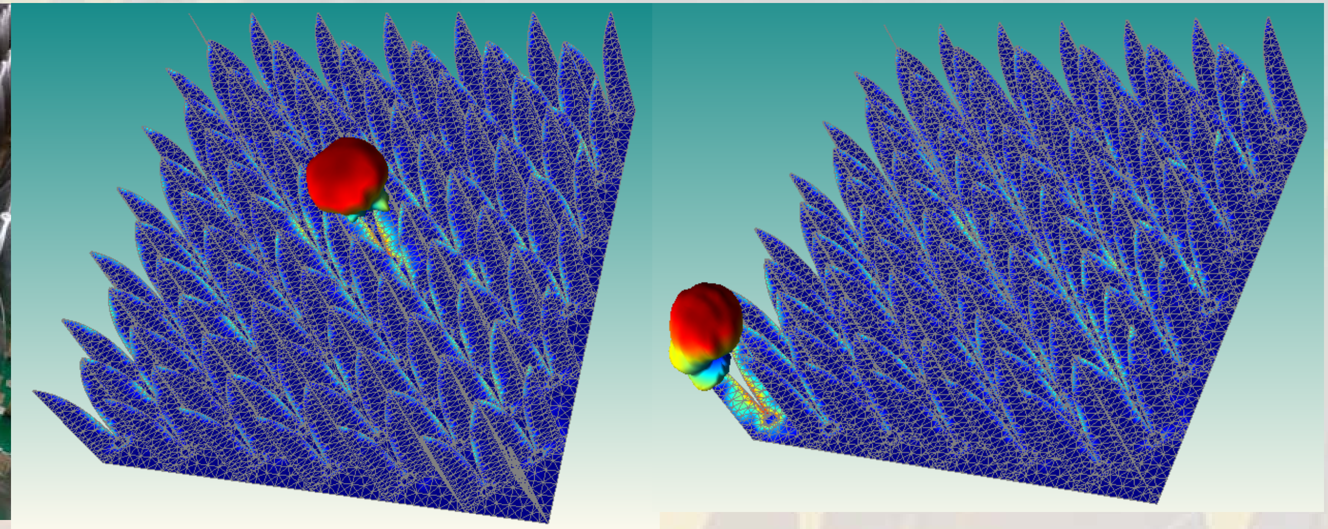
$$\text{XPD}_{\text{u}} = \frac{0.5^2}{0.5^2} = 1$$

$$\text{IXR}_{\text{J}} = \infty$$

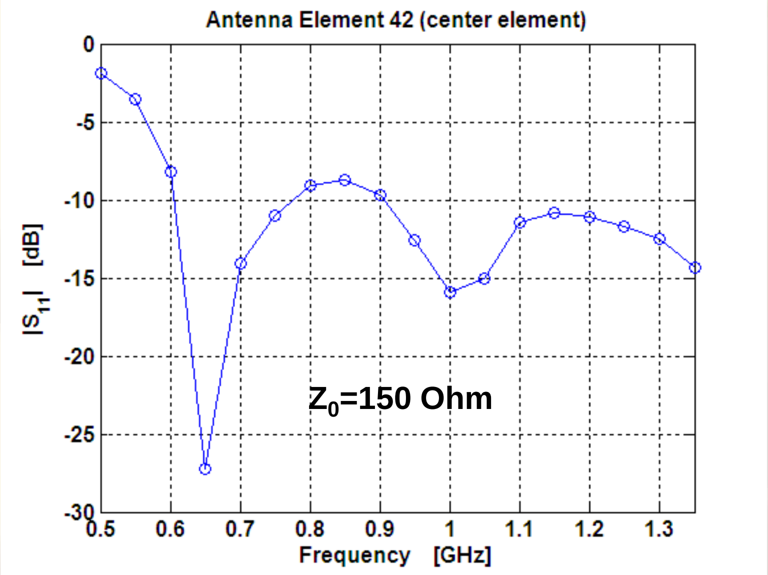
IXR measures the independency of a 2-D basis, XPD does not.

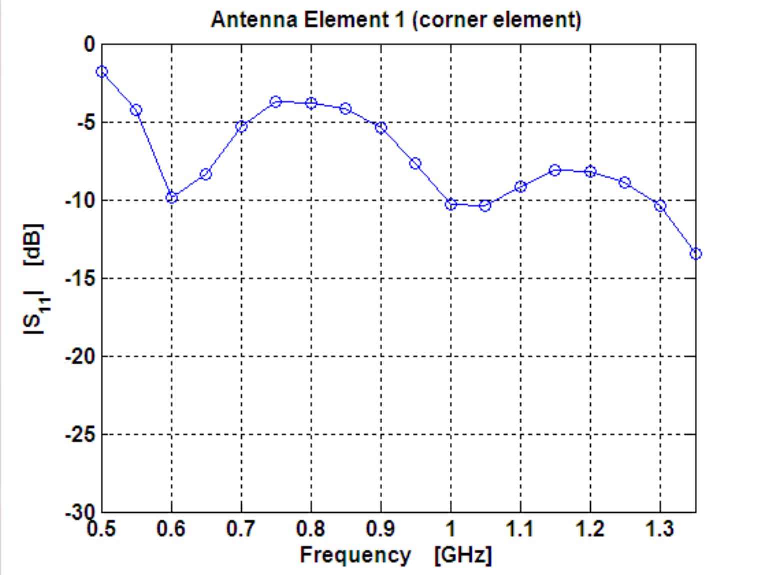
EMBRACE Tile

(all 72x72 TSA antenna elements used in simulations)

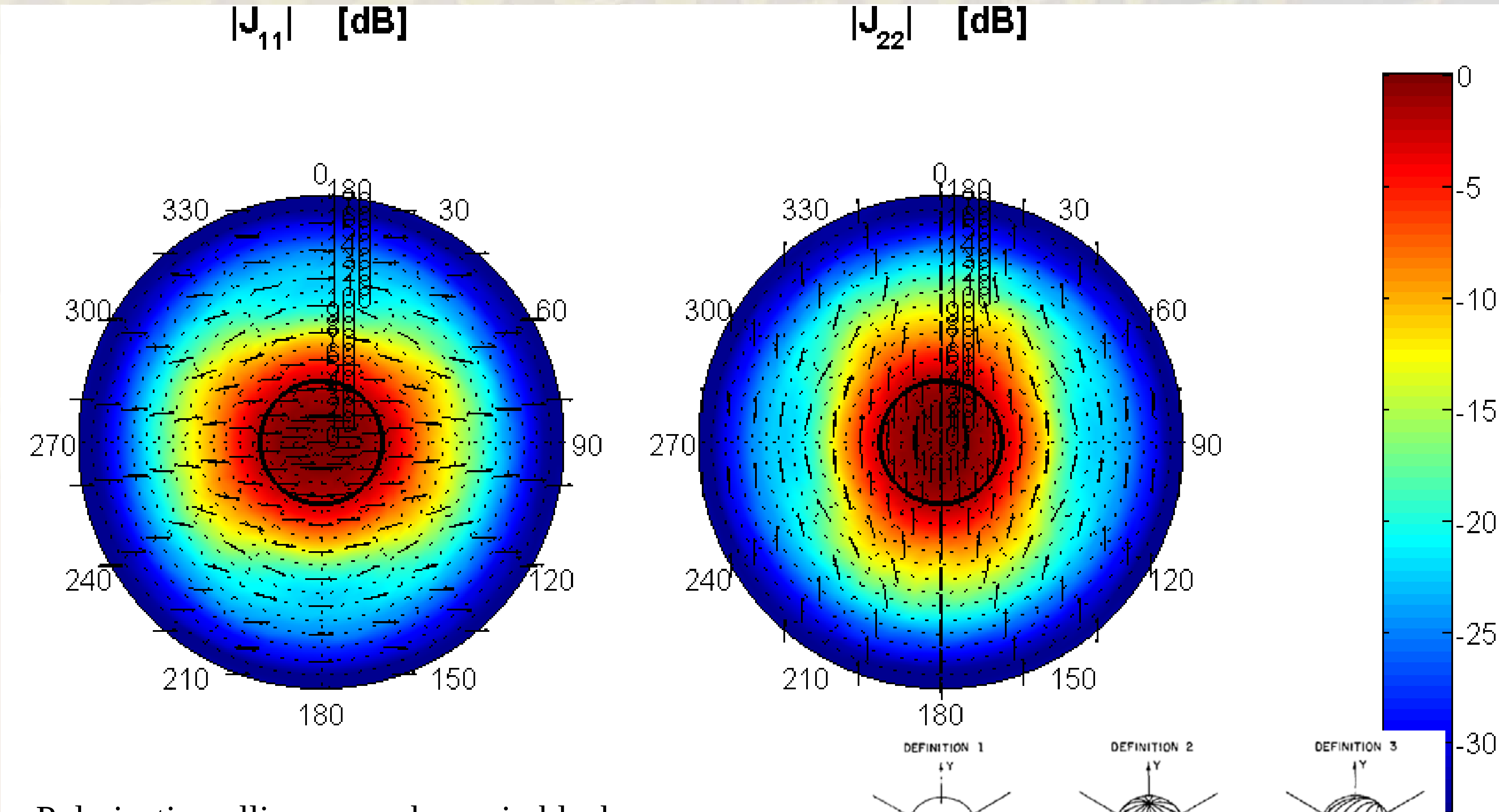


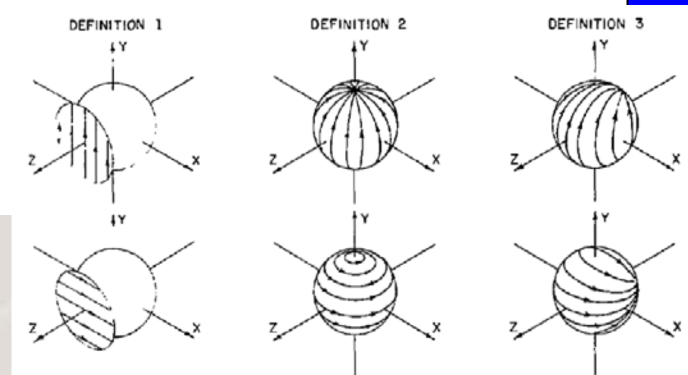
CBFM: - from 110833 RWG basis functions down to 5560 CBFs
- total execution time (single core) is 234 min. 13 sec.





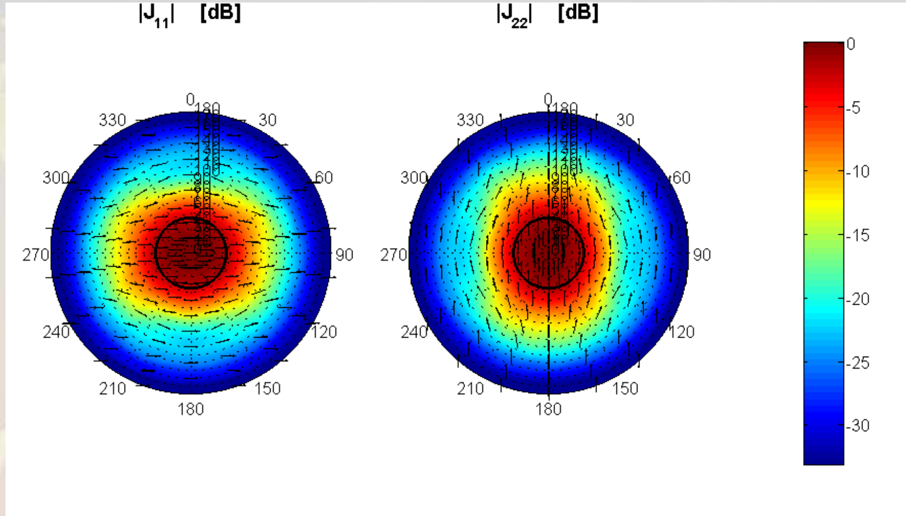
$|J_{11}|$ and $|J_{22}|$ as a function of angle (sampled at the beam centers)





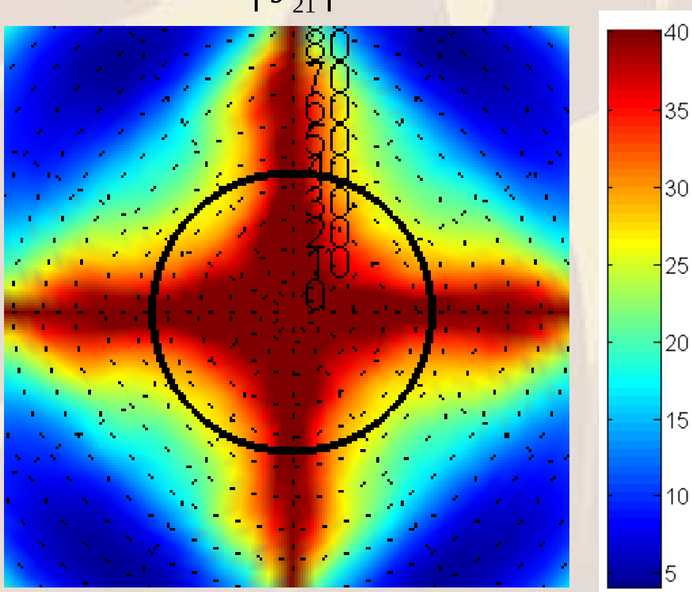
- Polarization ellipses are shown in black
- $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$ chosen according to Ludwig's 3rd definition
- Weights are chosen to receive maximum power (CFM)

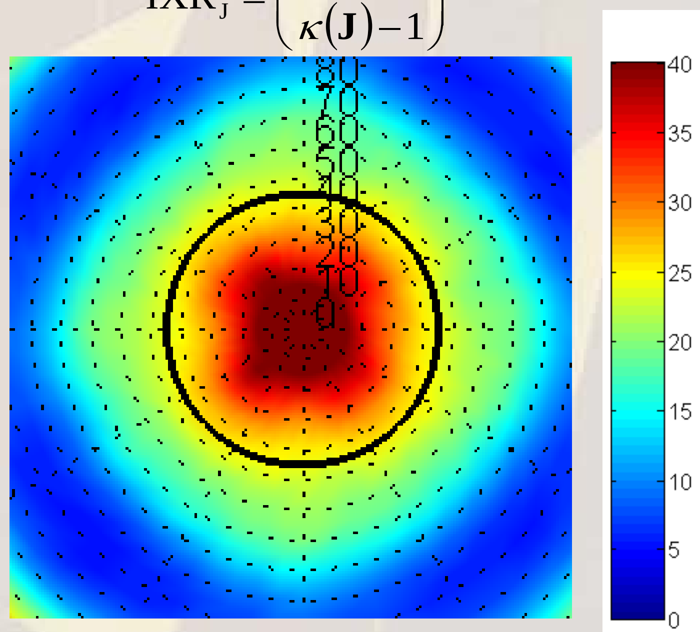
Simulated XPD and IXR



$$\mathrm{XPD}_u = \frac{|J_{11}|^2}{|J_{21}|^2} \quad [\mathrm{dB}]$$

$$\mathrm{IXR}_\mathrm{J} = \left(\frac{\kappa(\mathbf{J})+1}{\kappa(\mathbf{J})-1}\right)^2$$



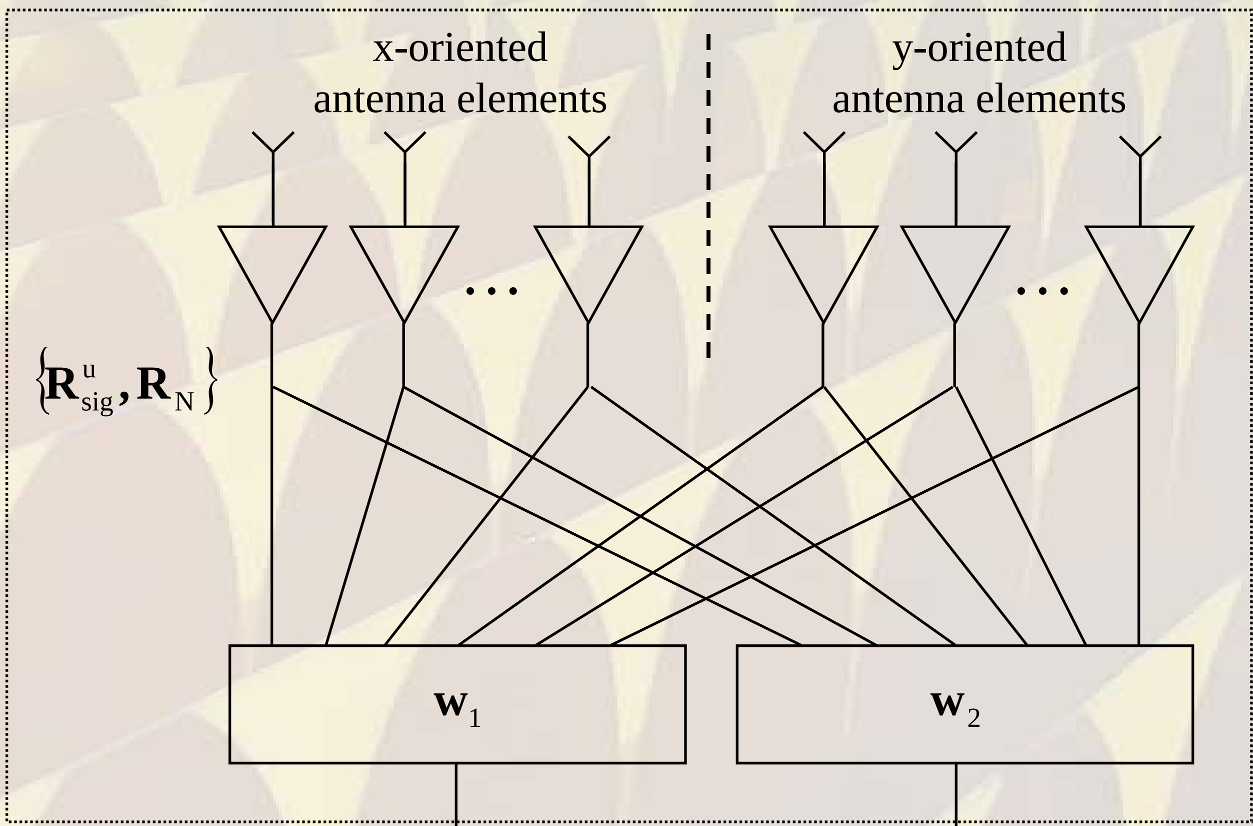


Joint optimization for both SNR and Polarization Purity

$$\left(E_{\mathrm{u}} = 1,\, E_{\mathrm{v}} = 0\right)$$

$$\boldsymbol{E}^{\mathrm{inc}} = \hat{\mathbf{u}}$$

deterministic CW signal



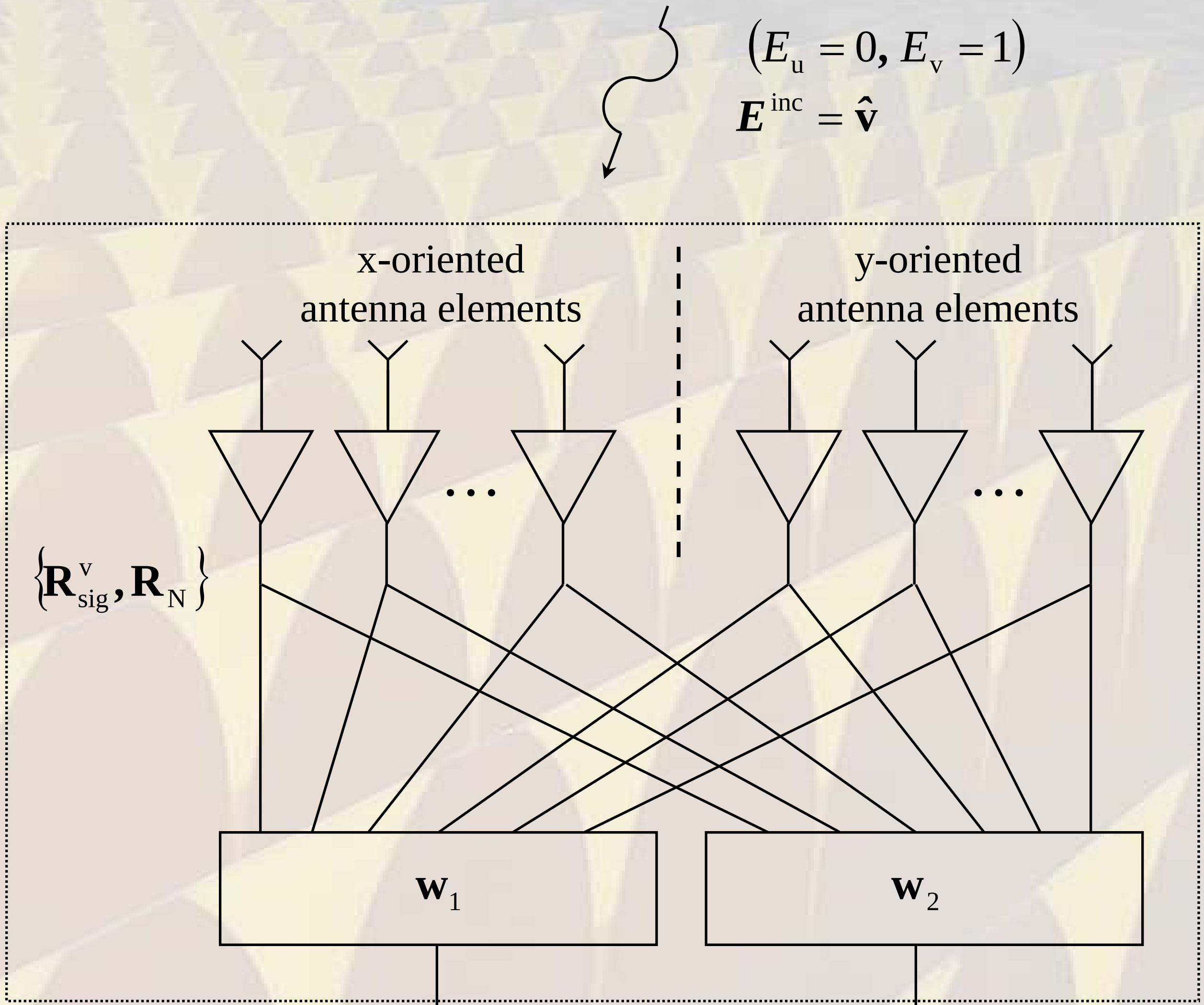
$$\left(\frac{S}{N}\right)_{11} = \frac{\mathbf{w}_1^H \mathbf{R}_{\mathrm{sig}}^{\mathrm{u}} \mathbf{w}_1}{\mathbf{w}_1^H \mathbf{R}_{\mathrm{N}} \mathbf{w}_1}$$

SNR self term

$$\left(\frac{S}{N}\right)_{21} = \frac{\mathbf{w}_2^H \mathbf{R}_{\mathrm{sig}}^{\mathrm{u}} \mathbf{w}_2}{\mathbf{w}_2^H \mathbf{R}_{\mathrm{N}} \mathbf{w}_2}$$

SNR leakage (cross) term

Joint optimization for both SNR and Polarization Purity



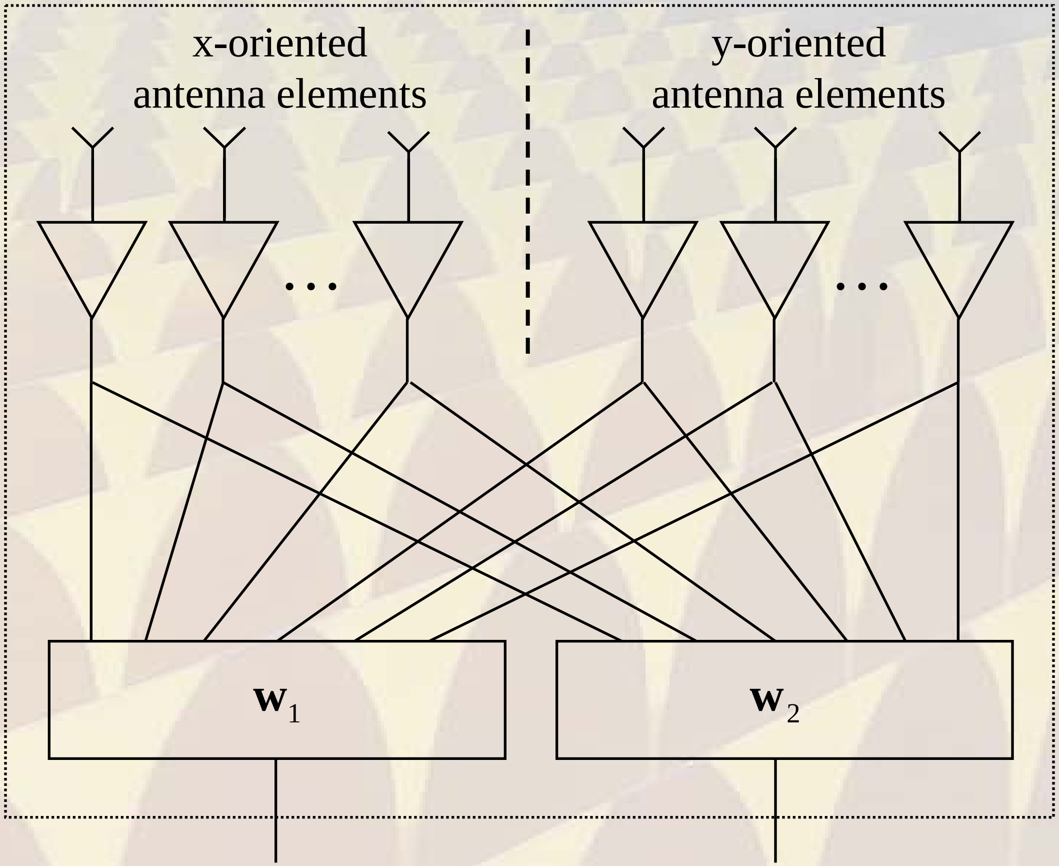
$$\left(\frac{S}{N}\right)_{12} = \frac{\mathbf{w}_1^H \mathbf{R}_{\mathrm{sig}}^{\mathrm{v}} \mathbf{w}_1}{\mathbf{w}_1^H \mathbf{R}_{\mathrm{N}} \mathbf{w}_1}$$

SNR leakage (cross) term

$$\left(\frac{S}{N}\right)_{22} = \frac{\mathbf{w}_2^H \mathbf{R}_{\mathrm{sig}}^{\mathrm{v}} \mathbf{w}_2}{\mathbf{w}_2^H \mathbf{R}_{\mathrm{N}} \mathbf{w}_2}$$

SNR self term

Joint optimization for both SNR and Polarization Purity



K. Warnick proposes a constraint optimization to determine the optimal weights: $\mathbf{J} = \mathbf{I}$ (calibrated system), then minimize noise

Calibrated for what reference coordinate system $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$?

B. Jeffs proposes to consider the leakage term as an additional noise term in the max SNR optimization:

$$\max\left(\frac{\mathbf{w}_1^H \mathbf{R}_{\mathrm{sig}}^{\mathrm{u}} \mathbf{w}_1}{\mathbf{w}_1^H \left(\mathbf{R}_{\mathrm{N}} + \mathbf{R}_{\mathrm{sig}}^{\mathrm{v}}\right) \mathbf{w}_1}\right)$$

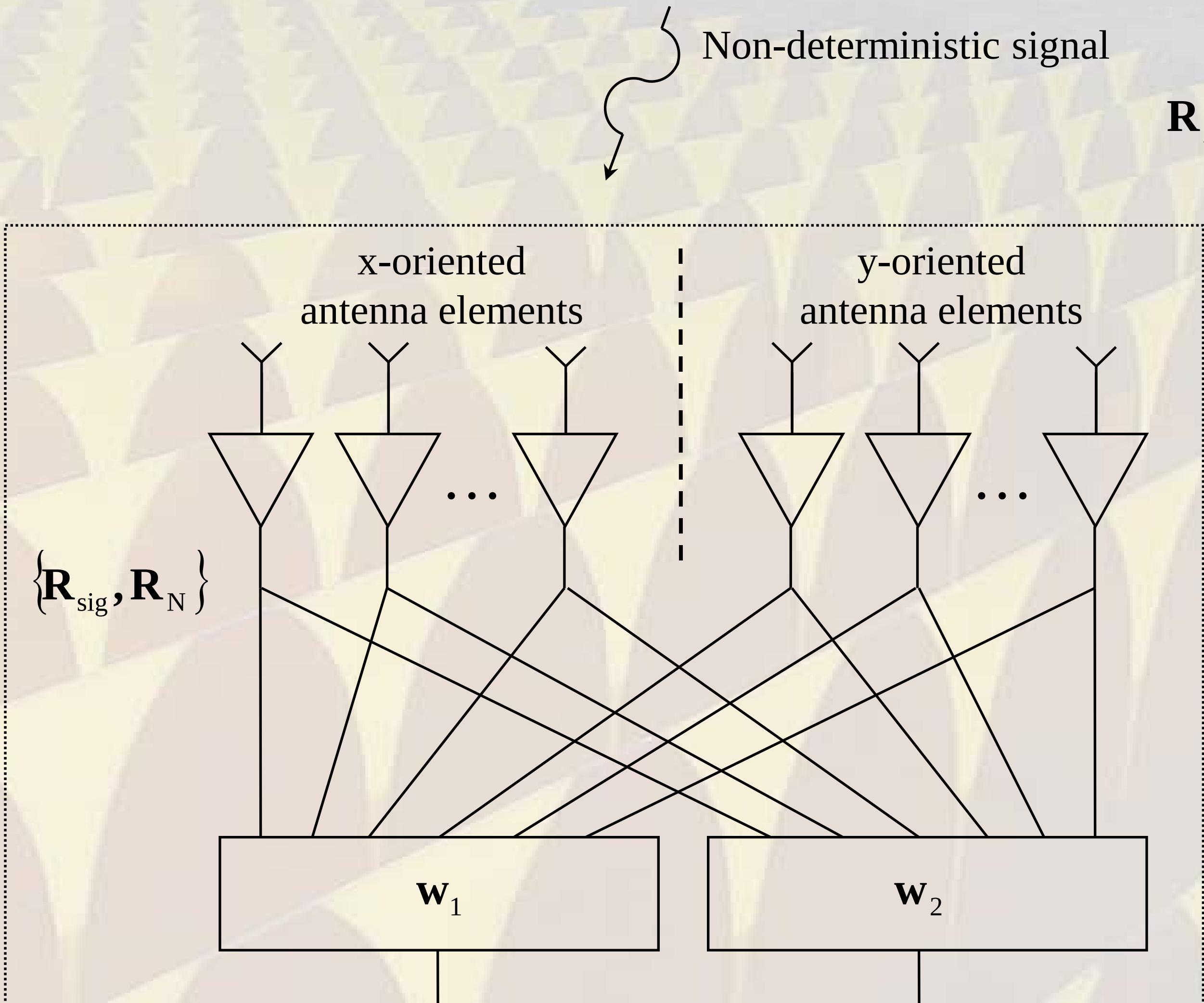
Weights depends on the source strength

Minimize condition number κ of the **SNR** matrix to maximize the sensitivity of the polarization measurement

$$\mathbf{SNR} = \begin{bmatrix} \left(\frac{S}{N}\right)_{11} & \left(\frac{S}{N}\right)_{12} \\ \left(\frac{S}{N}\right)_{21} & \left(\frac{S}{N}\right)_{22} \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{w}_1^H \mathbf{R}_{\mathrm{sig}}^{\mathrm{u}} \mathbf{w}_1}{\mathbf{w}_1^H \mathbf{R}_{\mathrm{N}} \mathbf{w}_1} & \frac{\mathbf{w}_1^H \mathbf{R}_{\mathrm{sig}}^{\mathrm{v}} \mathbf{w}_1}{\mathbf{w}_1^H \mathbf{R}_{\mathrm{N}} \mathbf{w}_1} \\ \frac{\mathbf{w}_2^H \mathbf{R}_{\mathrm{sig}}^{\mathrm{u}} \mathbf{w}_2}{\mathbf{w}_2^H \mathbf{R}_{\mathrm{N}} \mathbf{w}_2} & \frac{\mathbf{w}_2^H \mathbf{R}_{\mathrm{sig}}^{\mathrm{v}} \mathbf{w}_2}{\mathbf{w}_2^H \mathbf{R}_{\mathrm{N}} \mathbf{w}_2} \end{bmatrix}$$

$$\min(\kappa(\mathbf{SNR})) \rightarrow \{\mathbf{w}_1, \mathbf{w}_2\}$$

Joint optimization for both SNR and Polarization Purity



$$\left(\frac{S}{N}\right)_{11} = \frac{\mathbf{w}_1^H \mathbf{R}_{\mathrm{sig}} \mathbf{w}_1}{\mathbf{w}_1^H \mathbf{R}_{\mathrm{N}} \mathbf{w}_1} \qquad \left(\frac{S}{N}\right)_{22} = \frac{\mathbf{w}_2^H \mathbf{R}_{\mathrm{sig}} \mathbf{w}_2}{\mathbf{w}_2^H \mathbf{R}_{\mathrm{N}} \mathbf{w}_2}$$

$$\mathbf{R}_E = \begin{bmatrix} \overline{|E_{\mathrm{u}}|^2} & \overline{E_{\mathrm{u}}(E_{\mathrm{v}})^*} \\ \overline{E_{\mathrm{v}}(E_{\mathrm{u}})^*} & \overline{|E_{\mathrm{v}}|^2} \end{bmatrix}$$

$\mathbf{R}_{\mathrm{sig}}$ has a rank of two for partially polarized waves, with eigenvectors: $\mathbf{e}_1$ and $\mathbf{e}_2$

Bruce Veidt’s method:
(based on max. SNR method)

$$\mathbf{w}_1 = \mathbf{R}_{\mathrm{N}}^{-1}\mathbf{e}_1$$

$$\mathbf{w}_2 = \mathbf{R}_{\mathrm{N}}^{-1}\mathbf{e}_2$$

For unpolarized source, the realized output voltage vectors can have arbitrary rotation, so calibration is still required?

Open Questions

- Which pair of weight vectors have to be chosen to simultaneously optimize for good polarization characteristics and high SNR? Maybe we need to look into robust beamformer algorithms that are insensitive to first-order perturbations in our system?

- Does Bruce Veidt's method yield good and stable polarimetric properties for each beam? How important is polarization stability over the FoV before calibration?

- How does an error on the Stokes parameters translate into requirements for the XPD, XPI, and IXR?

- What reference frame should be taken for $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$? (this question may be of less importance when all the antenna elements are beam formed, otherwise the intrinsic polarization properties must be good enough)