Comparison of tuner-based noise-parameter extraction methods for measurements of room-temperature SKA LNAs

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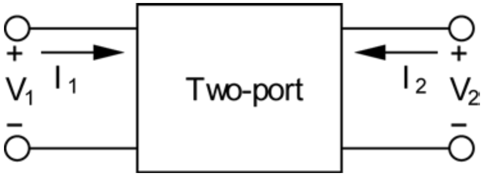
Outline

- Review: Two-port networks
- Noise-parameter measurement
- Equipment measurement accuracy
- Monte Carlo analysis
- Experimental results
- SKA LNA example
- Conclusions

Review: Two-port networks

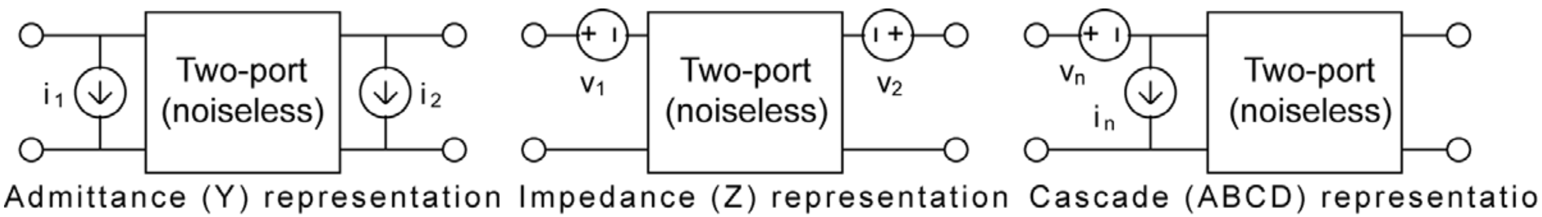
- A single-ended low noise amplifier (LNA) can be represented by a two-port network



- The electrical performance of two-port is analyzed with:
  - Z-parameters $([V]=[Z][I])$
  - Y-parameters $([I]=[Y][V])$
  - S-parameters $([b]=[s][a]$ where a is the incident wave and b is the reflected wave$)$
  - ABCD parameters $\left(\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ I_2 \end{bmatrix}\right)$
  - Etc.
- These electrical parameters are represented by 2x2 matrices

Review: Two-port networks

- Similarly to electrical parameters, noise parameters of two-ports can be expressed with 2x2 matrices.
- The noisy two port networks are represented by a noiseless two-port network (discussed above) and two noise sources.

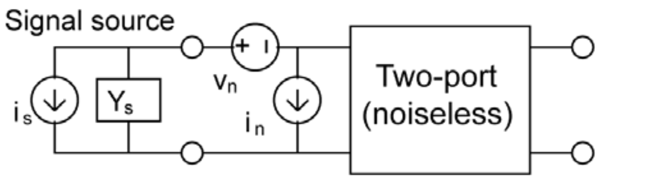


- The noise sources may be correlated

$$C_Y = \frac{1}{2\Delta f}\begin{vmatrix} i_1 i_1^* & i_1 i_2^* \\ i_1^* i_2 & i_2 i_2^* \end{vmatrix} \qquad C_Z = \frac{1}{2\Delta f}\begin{vmatrix} v_1 v_1^* & v_1 v_2^* \\ v_1^* v_2 & v_2 v_2^* \end{vmatrix} \qquad C_{ABCD} = \frac{1}{2\Delta f}\begin{vmatrix} v_n v_n^* & v_n i_n^* \\ v_n^* i_n & i_n i_n^* \end{vmatrix}$$

Review: Noisy two-port networks

- <u>Why does one need the noise correlation matrices?</u>
  - Consider a two-port network driven by a signal source



$$C_{ABCD} = \frac{1}{2\Delta f}\begin{vmatrix} v_n v_n^* & v_n i_n^* \\ v_n^* i_n & i_n i_n^* \end{vmatrix} = 2kT\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix}$$

  - The noise figure (factor) of this two-port is:

$$F = \frac{i_s^2 + \left|i_n + Y_s v_n\right|^2}{i_s^2} = 1 + \frac{\left(i_n + Y_s v_n\right)\left(i_n + Y_s v_n\right)^*}{i_s^2} = 1 + \frac{i_n i_n^* + Y_s v_n i_n^* + Y_s^* i_n v_n^* + Y_s Y_s^* v_n v_n^*}{i_s^2}$$

and since $i_s^2 = 4kT\Delta f\ \mathrm{Re}\{Y_s\}$ then $F = 1 + \dfrac{c_{22} + Y_s c_{12} + Y_s^* c_{21} + \left|Y_s\right|^2 c_{11}}{\mathrm{Re}\{Y_s\}}$

  - For a given Y_s, the noise figure of a two-port network is obtained from C_{ABCD}! Nothing else is required!

Review: Two-port networks

- The noise figure (factor) of this two-port was found to be:

$$F = 1 + \frac{c_{22} + Y_s c_{12} + Y_s^* c_{21} + \left|Y_s\right|^2 c_{11}}{\mathrm{Re}\{Y_s\}}$$

- Can solve for $Y_{s,opt} = G_{s,opt} + jB_{s,opt}$ that minimizes the noise figure

$$Y_{s,opt} = \sqrt{\frac{c_{22}}{c_{11}} - \left(\frac{\mathrm{Im}\{c_{12}\}}{c_{11}}\right)^2} + j\,\frac{\mathrm{Im}\{c_{12}\}}{c_{11}}$$

- The lowest (minimum) noise figure is obtained by substituting $Y_{s,opt}$ back into F:

$$F_{\min} = 1 + \frac{c_{22} + Y_{s,opt} c_{12} + Y_{s,opt}^* c_{21} + \left|Y_{s,opt}\right|^2 c_{11}}{\mathrm{Re}\{Y_{s,opt}\}} = 1 + 2\left(\mathrm{Re}\{c_{12}\} + G_{s,opt} c_{11}\right)$$

- If $Y_s \neq Y_{s,opt}$ then the noise figure (factor) is $F = F_{\min} + \frac{c_{11}}{\mathrm{Re}\{Y_s\}}\left|Y_s - Y_{s,opt}\right|^2$

Review: Two-port networks

- $Y_{s,opt}$, F_{min}, and F can be found directly by using the noise correlation matrix:

$$C_{ABCD} = 2kT\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} \qquad Y_{s,opt} = \sqrt{\frac{c_{22}}{c_{11}} - \left(\frac{\mathrm{Im}\{c_{12}\}}{c_{11}}\right)^2} + j\,\frac{\mathrm{Im}\{c_{12}\}}{c_{11}}$$

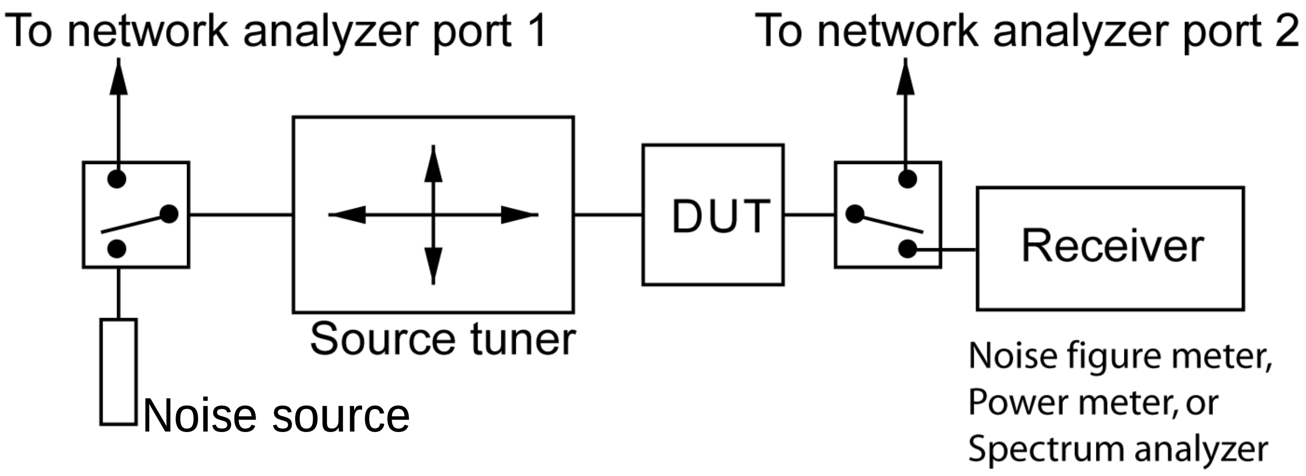
$$F_{\min} = 1 + 2\left(\mathrm{Re}\{c_{12}\} + G_{s,opt}c_{11}\right) \qquad F = F_{\min} + \frac{c_{11}}{\mathrm{Re}\{Y_s\}}\left|Y_s - Y_{s,opt}\right|^2$$

- Commonly $Y_{s,opt}$, F_{min}, and $R_n = c_{11}$ are called the noise parameters of a two-port network.
- In terms of noise parameters, the noise correlation matrix is

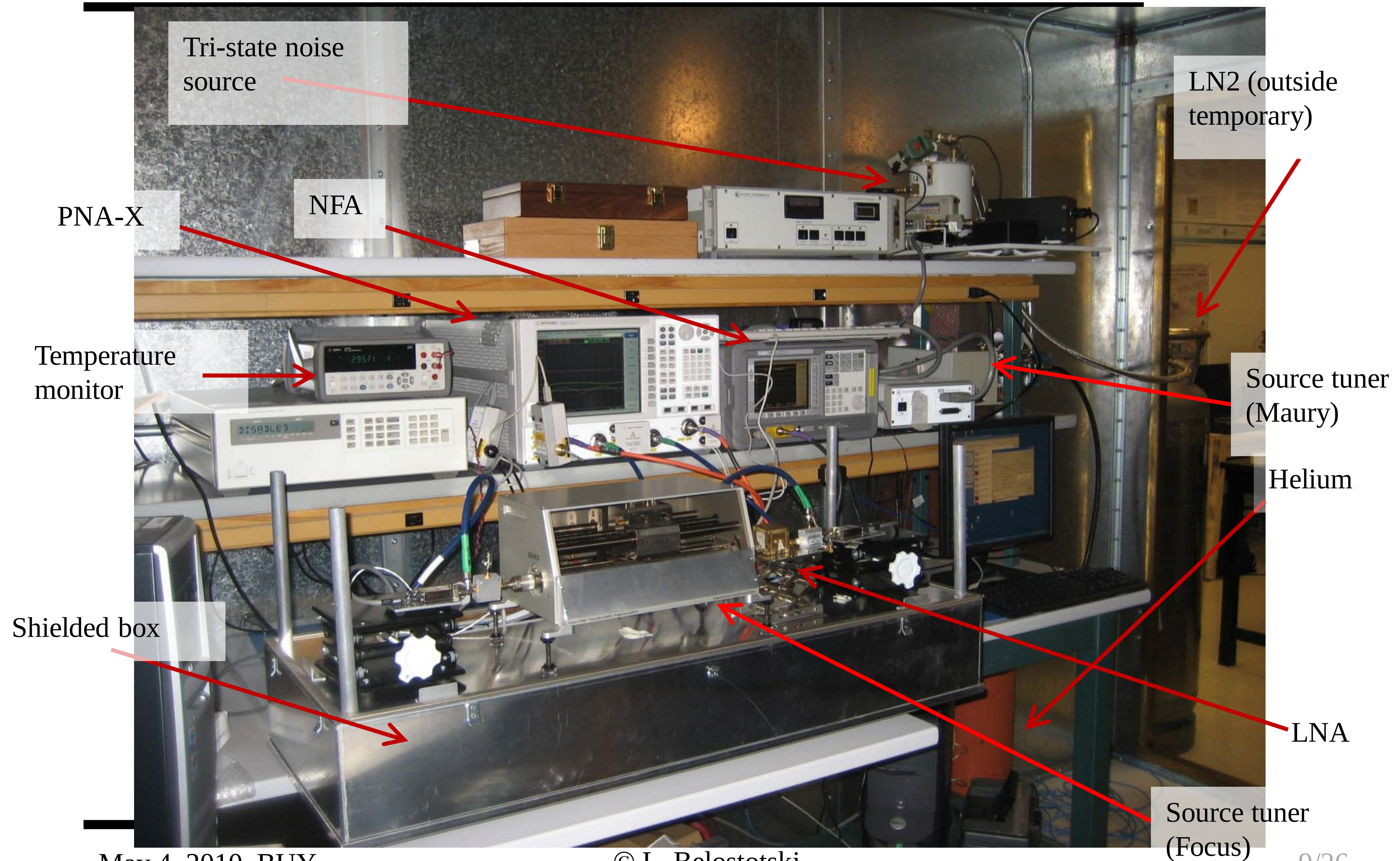
$$C_{ABCD} = 2kT\begin{vmatrix} R_n & \frac{F_{\min}-1}{2} - R_n Y_{s,opt}^* \\ \frac{F_{\min}-1}{2} - R_n Y_{s,opt} & R_n\left|Y_{s,opt}\right|^2 \end{vmatrix}$$

Noise-parameter measurement

- There are a few techniques: tuner based, mismatched line based, and noise wave-amplitude based, etc
- This talk focus on tuner-based extraction techniques



Noise-parameter measurement

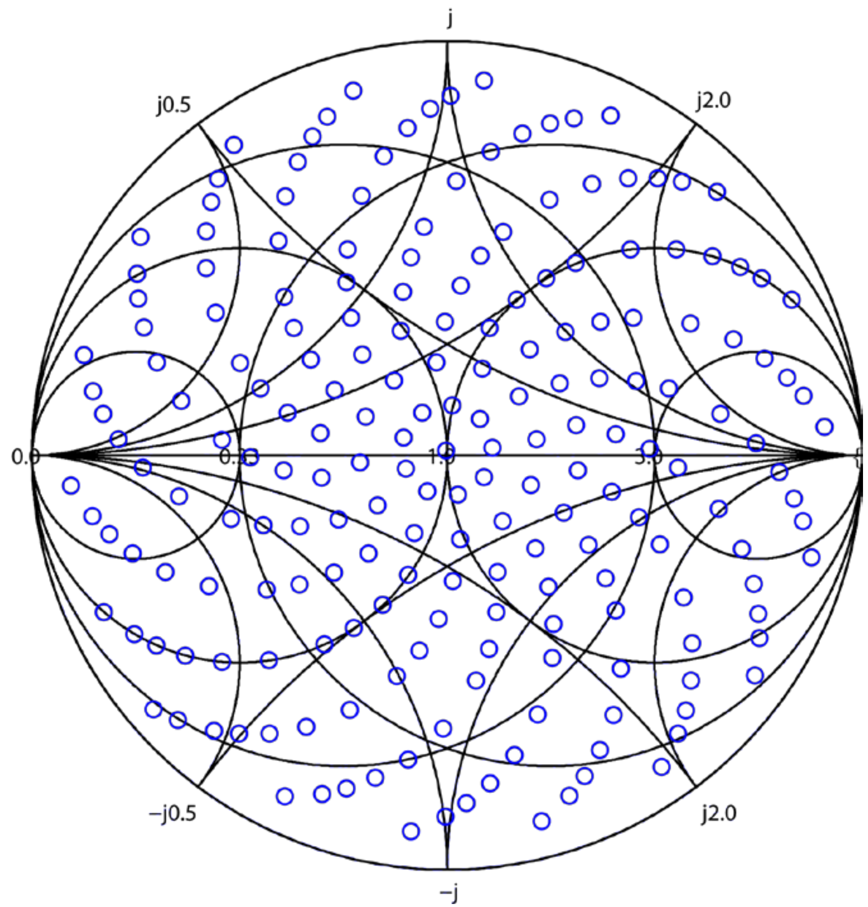


Noise-parameter measurement

- Tuner can generates a number of impedances (admittances) at the receiver (or the DUT) input
- Remember

$$F = F_{\min} + \frac{R_n}{G_s}\left|Y_{opt} - Y_s\right|^2$$

- Measurement of noise figures at a few of these admittances allows the extraction of (F_{min}, R_n and Y_{opt})



Noise-parameter measurement

- How are the noise figures measured?
- Two approaches:
  - Cold methods
  - Hot-cold methods (aka modified Y-factor methods)

Noise-parameter measurement (Cold methods)

- Cold methods:
  - –Noise source <u>only</u> toggles from OFF to ON when tuner synthesizes 50Ohm impedance and the receiver measures two noise power levels

$$\begin{cases} P_{\mathrm{cal}}^{c} = \left(P_{\mathrm{in}}^{c} + \frac{T_{a,0}^{c}}{T_0} P_{\mathrm{rec,cal}}^{c}\right) M^c G_{\mathrm{rec}} \\ P_{\mathrm{cal}}^{h} = \left(P_{\mathrm{in}}^{h} + \frac{T_{a,0}^{h}}{T_0} P_{\mathrm{rec,cal}}^{h}\right) M^h G_{\mathrm{rec}}, \end{cases} \qquad P_{\mathrm{in}}^{c(h)} = kB \underbrace{\left[T_{\mathrm{ns}}^{c(h)} + T_{i\mathrm{Net}}^{c(h)}\right] G_{\mathrm{A},i\mathrm{Net}}^{c(h)}}_{T_{\mathrm{in}}^{c(h)}}$$

where $M^{c(h)}G_{rec}$ is the receiver transducer gain, $P^{c(h)}_{in}$ noise power at the receiver input, $P^{c(h)}_{rec,cal}$ receiver noise power, and $T^{c(h)}_{iNet}$ and $G^{c(h)}_{A,iNet}$ are the noise temperature and available gain of the tuner.

  - – With a few different impedances at the receiver input, G_{rec} and the receiver noise parameters are obtained
  - –Note: noise source impedance changes when it toggles which complicates the extraction of G_{rec}.

Noise-parameter measurement (Cold methods)

- Based on the way G_{rec} is found, cold methods are subdivided into:
  - Simplified Cold method (assumes noise source impedance does not change when it toggles and assumes $T_a = T^c_{ns}$)
  - Iterative Cold method (uses iterative approach to finding G_{rec})
  - Direct Cold method (uses least-squares approach to find G_{rec} and the noise parameters simultaneously, similar to the modified Y-factor method)
- For DUT measurements the noise source is always OFF.
  - Noise powers measured by receiver are related to the DUT noise powers through G_{rec} .
  - Errors in G_{rec} cause errors in noise parameters.

Noise-parameter measurement (modified Y-factor)

- Modified Y-factor method (Hot-Cold method):
  - Noise source toggles from OFF to ON at impedance synthesized by the tuner
  - The receiver measures two noise power levels for each tuner impedance

$$\begin{cases} P^c_{\mathrm{cal}} = \left(P^c_{\mathrm{in}} + \frac{T^c_{a,0}}{T_0} P^c_{\mathrm{rec,cal}}\right) M^c G_{\mathrm{rec}} \\ P^h_{\mathrm{cal}} = \left(P^h_{\mathrm{in}} + \frac{T^h_{a,0}}{T_0} P^h_{\mathrm{rec,cal}}\right) M^h G_{\mathrm{rec}} \end{cases}$$

  - G_{rec} and the noise parameters are determined using a least-squares fit
  - For each impedance synthesized by the tuner, G_{rec} is indirectly measured → expect better accuracy in extracting G_{rec}.

Equipment measurement accuracy

MEASUREMENT UNCERTAINTIES ADDED TO EVALUATE NOISE PARAMETER EXTRACTION METHODS BASED ON THE INSTRUMENT SPECIFICATIONS NOTED IN THE REFERENCE COLUMN

Measured parameter	Uncertainty	Noise model	Note	References
S-parameters	mean=-65 dB max=-60 dB	Rayleigh	High power, Notes 1,2	[60]
	mean=-55 dB max=-50 dB	Rayleigh	Low power, Notes 1,2,3	
Tuner repeatability	mean=-50 dB max=-44 dB	Rayleigh	Notes 4,5	[61]–[63]
T_c measurement accuracy	mean=$0.25\,^\circ$C st. dev.=$0.33\,^\circ$C	Gaussian	Note 6	[64]
Receiver accuracy	0.017 dB	Gaussian	Note 7	[65]
Noise source ENR	0.134 dB	Constant offset	Note 8	[64]

Note 1: Since the uncertainties in the real and imaginary parts of S-parameter measurements are correlated by an unknown amount [66], the uncorrelated real and imaginary parts of S-parameters were selected as they produce larger errors in the extracted noise parameters
Note 2: Values are based on the authors calibration experience with TRL kits using 128 averages and [60]
Note 3: When measuring the receiver input reflection coefficient and the DUT S-parameters, the VNA output power needs to be backed off in order to avoid compression of the two devices. Depending on linearity specifications of the DUT, power levels of less than -40 dBm are required
Note 4: It was assumed that phase errors and magnitude errors are not correlated
Note 5: Some automatic tuner vendors allow users to perform noise measurements at tuner locations that have not been calibrated but rather the supplied software estimates the actual reflection coefficient values based on measured surrounding points. Based on our measurements the estimated points worsen the tuner repeatability to a mean value of -35 dB with the maximum difference reaching -22 dB
Note 6: To assess the effect of errors in measurements of T_a and T_{ns}^c, a constant offset representing a reasonable accuracy of temperature measurements of 0.25 K was introduced for both temperature "readings" used by the Matlab program to extract the noise parameters
Note 7: When the noise figure analyzer bandwidth is 4MHz
Note 8: Average ENR uncertainty for an Agilent noise source N4000A in the authors' possession

Monte Carlo analysis

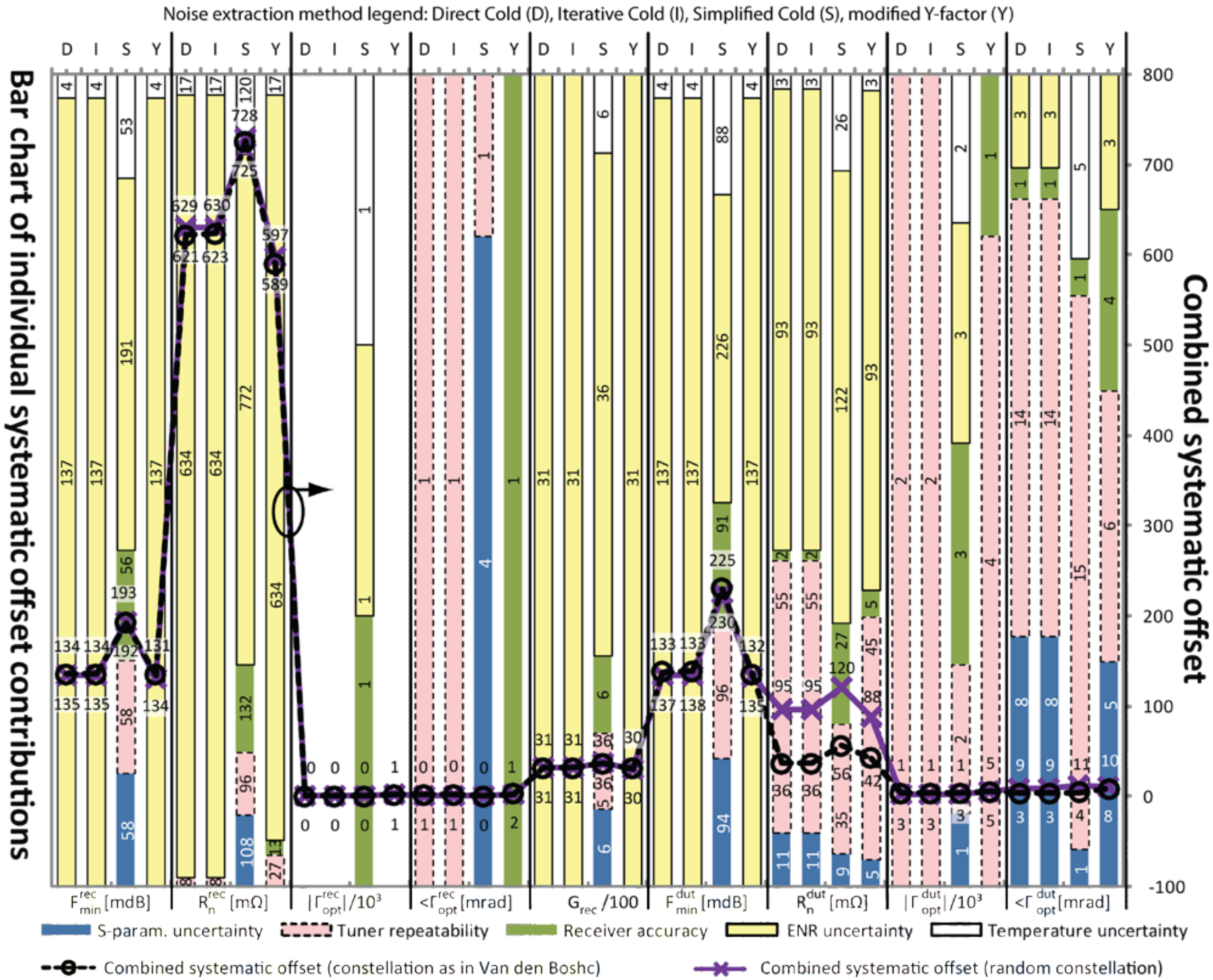
SIMULATION PARAMETERS

	Γ_{ns}^{c}	Γ_{ns}^{h}	ENR
Noise Source	$0.022\angle 144^\circ$	$0.019\angle 140^\circ$	5 dB

	Γ_{rec}	F_{min}	R_n	G_{opt}	G_{rec}
Receiver	$0.057\angle -136^\circ$	3 dB	$20\,\Omega$	$0.15\angle -95^\circ$	100

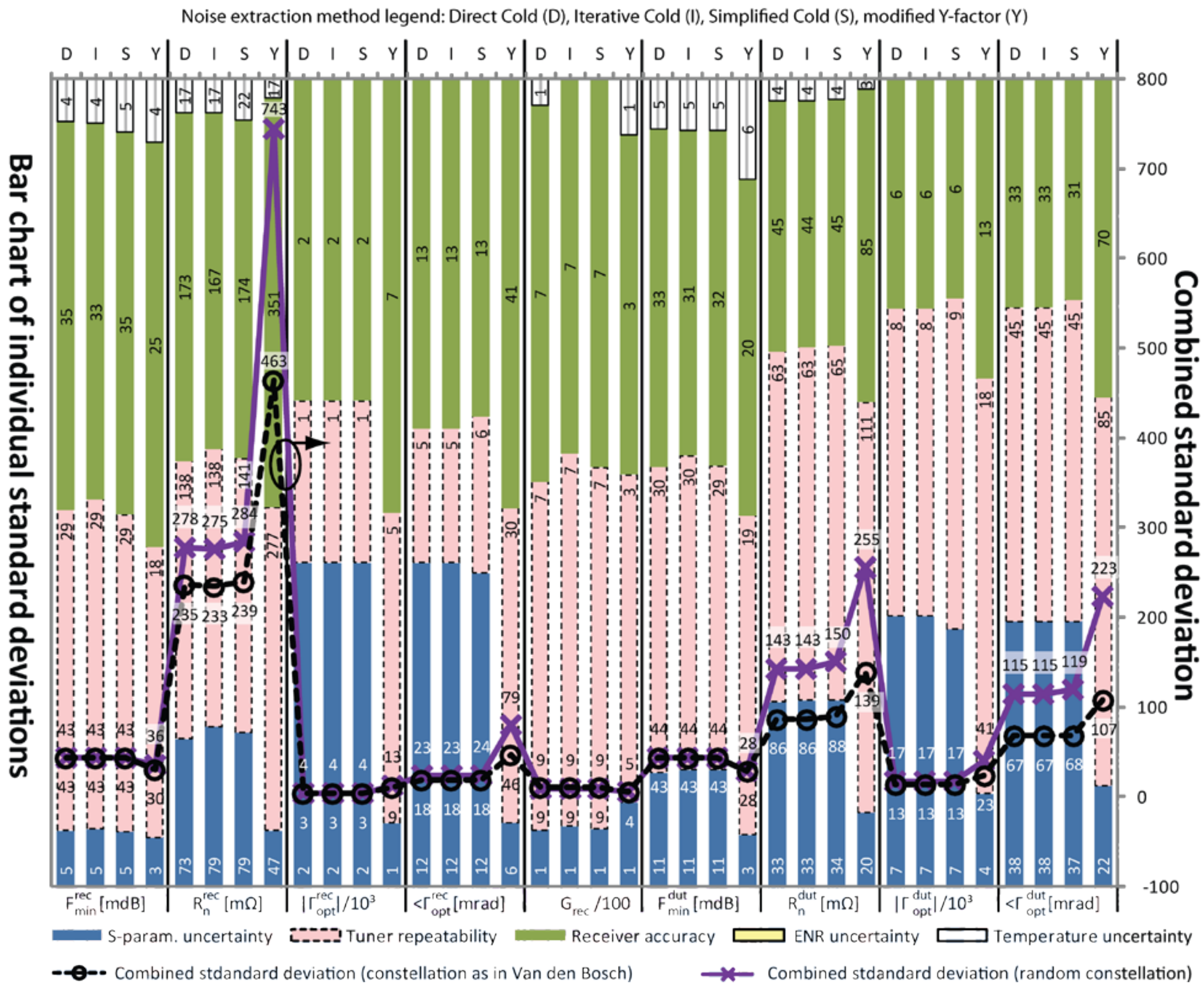
	S_{11}	S_{21}	S_{12}	S_{22}	F_{min}	R_n	G_{opt}
DUT	$0.3\angle 50^\circ$	$7.6\angle 37^\circ$	$0.032\angle 86^\circ$	$0.7\angle -93^\circ$	0.2 dB	$3\,\Omega$	$0.15\angle 55^\circ$

Monte Carlo analysis



- Simple Cold method gives the worst performance
- Modified Y-factor is the best for Rn extractions but not Γopt.

Monte Carlo analysis



- Modified Y-factor is the best for Fmin and Grec extractions but not Rn and Γopt.

Experimental results

RECEIVER NOISE PARAMETER EXTRACTION TESTS. DURING TEST 4, T_a WAS SET TO A CONSTANT 24 °C. THE NOISE FIGURE ANALYZER BANDWIDTH WAS SET TO 1 MHz

Test	Tuner calibration	Γ_{rec} meas.	Receiver ave.	T_a meas.	No of meas.	Constellation
1	Yes	Yes	32	Yes	25	Van den Bosch
2	No	No	32	Yes	25	Van den Bosch
3	No	No	128	Yes	25	Van den Bosch
4	No	No	32	No	25	Van den Bosch
5	No	No	32	Yes	25	all 217 tuner impedances
6	No	No	32	Yes	150	Van den Bosch

Experimental results

STANDARD DEVIATIONS OBTAINED FROM MEASUREMENTS DURING TESTS 1–6. THE NOISE FIGURE ANALYZER BANDWIDTH WAS SET TO 1 MHz

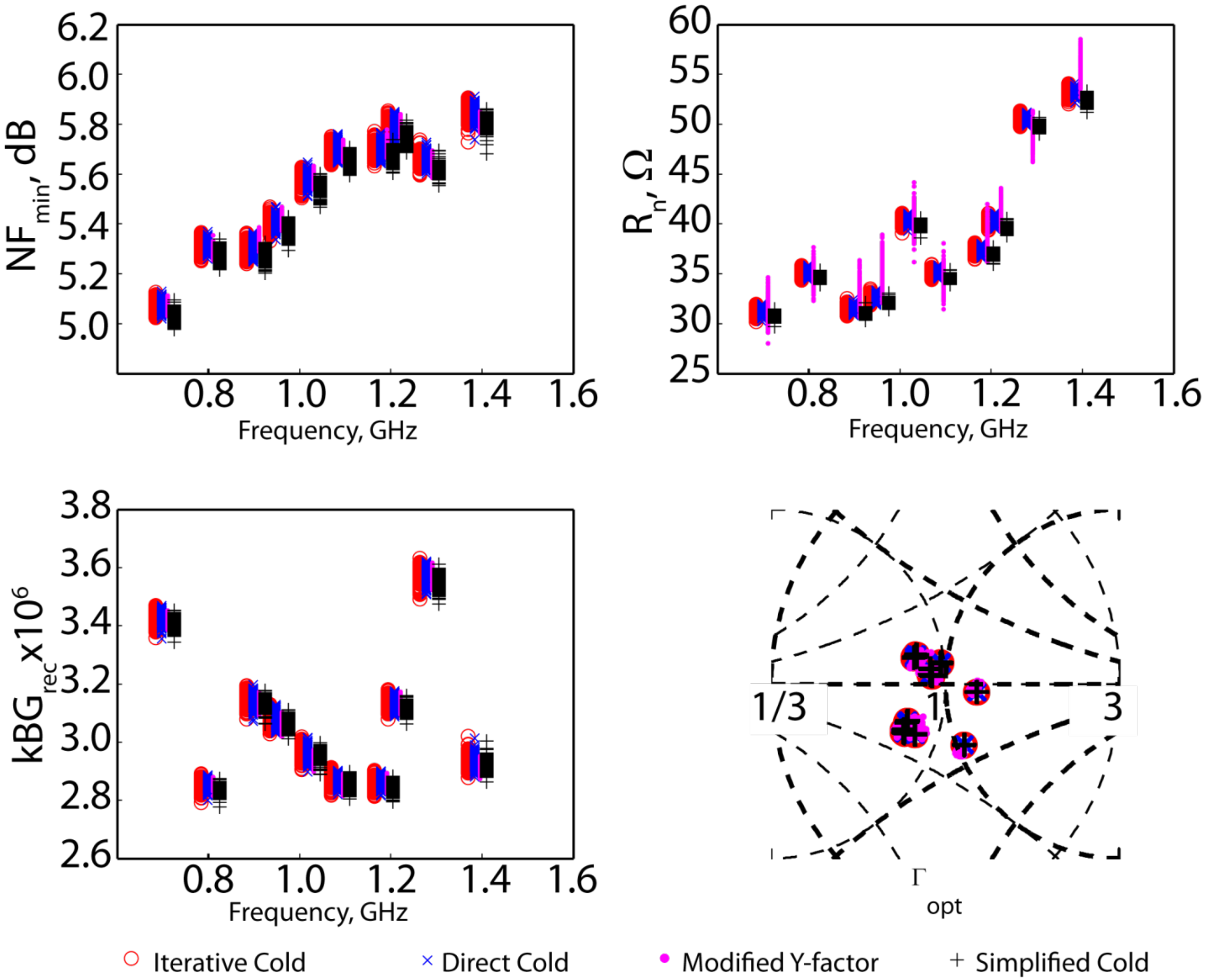
Test	T_{min}, K	R_n, Ω	$\lvert\Gamma_{opt}\rvert$	$\angle\Gamma_{opt}$	kBG_{rec}
1	21	0.32	0.0018	1.8°	2.3e-8
2	20	0.31	0. 0019	1.6°	1.7e-8
3	12	0.22	0.0017	1.0°	1.4e-8
4	20	0.31	0.0016	1.8°	2.0e-8
5	24	0.32	0.0031	1.3°	2.0e-8
6	18	0.31	0.0017	1.8°	1.8e-8

S-parameter measurements are good enough

NFA averaging is important!!!

Tuner repeatability is the most dominant source of error

Experimental results (150 measurements)



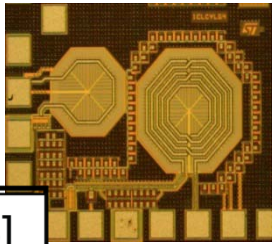
Experimental results (150 measurements)

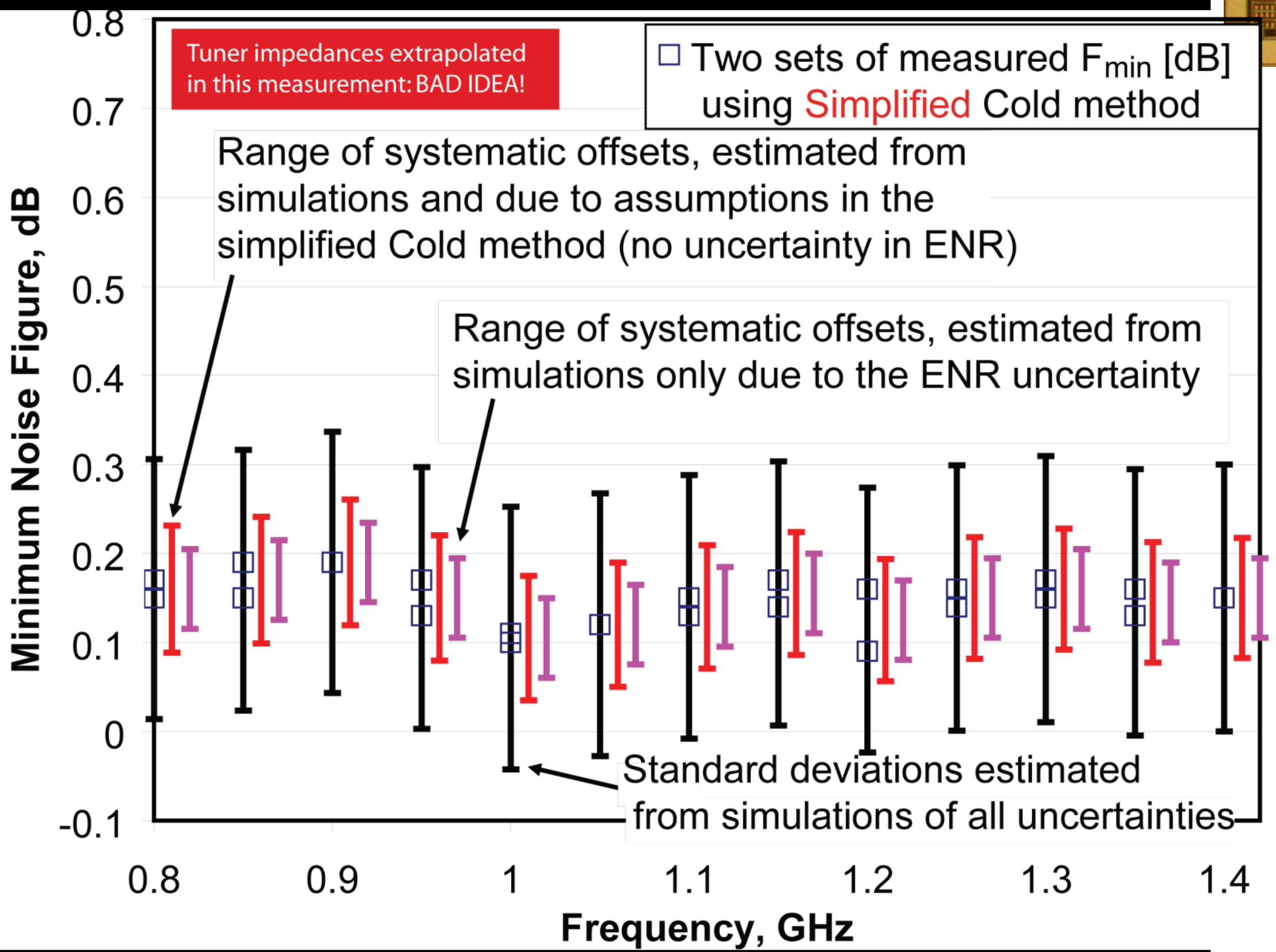
Comparison of Standard Deviations From the Four Noise Parameter Extraction Methods Normalized by the Corresponding Data Obtained With the Iterative Cold Method. The Noise Figure Analyzer Bandwidth Was Set to 1 MHz

	Direct Cold	Modified Y-factor	Simplified Cold
T_{min}	0.9	0.7	1.0
R_n	1.0	3.8	1.0
$\lvert\Gamma_{opt}\rvert$	1.0	4.4	1.0
$\angle\Gamma_{opt}$	1.0	5.0	1.0
kBG_{rec}	0.9	0.7	1.0

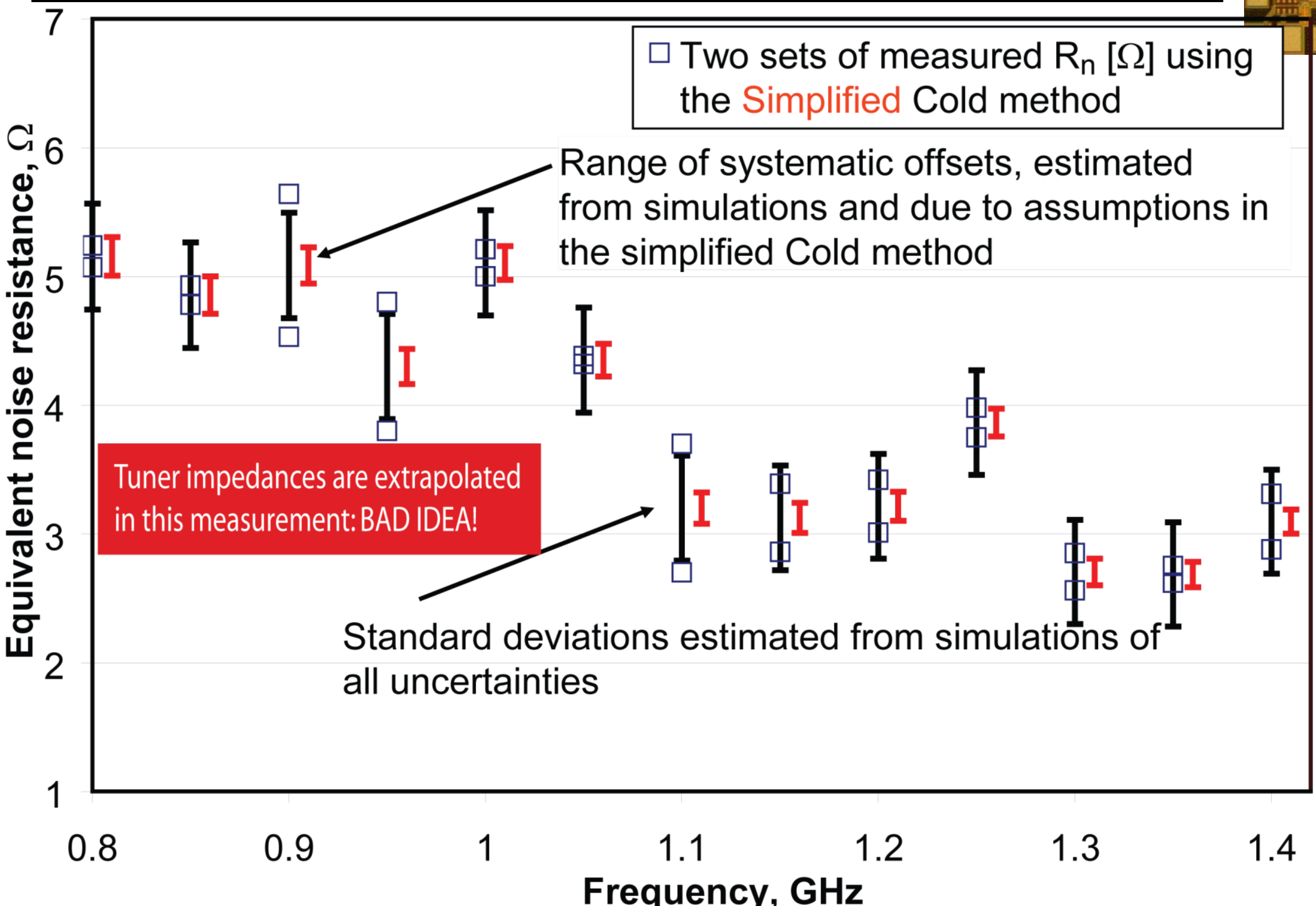
These numbers don’t capture systematic offsets seen in the previous slide. The simplified Cold method deviates the most from the other methods.

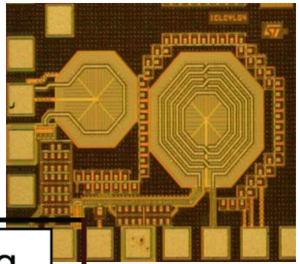
SKA LNA example



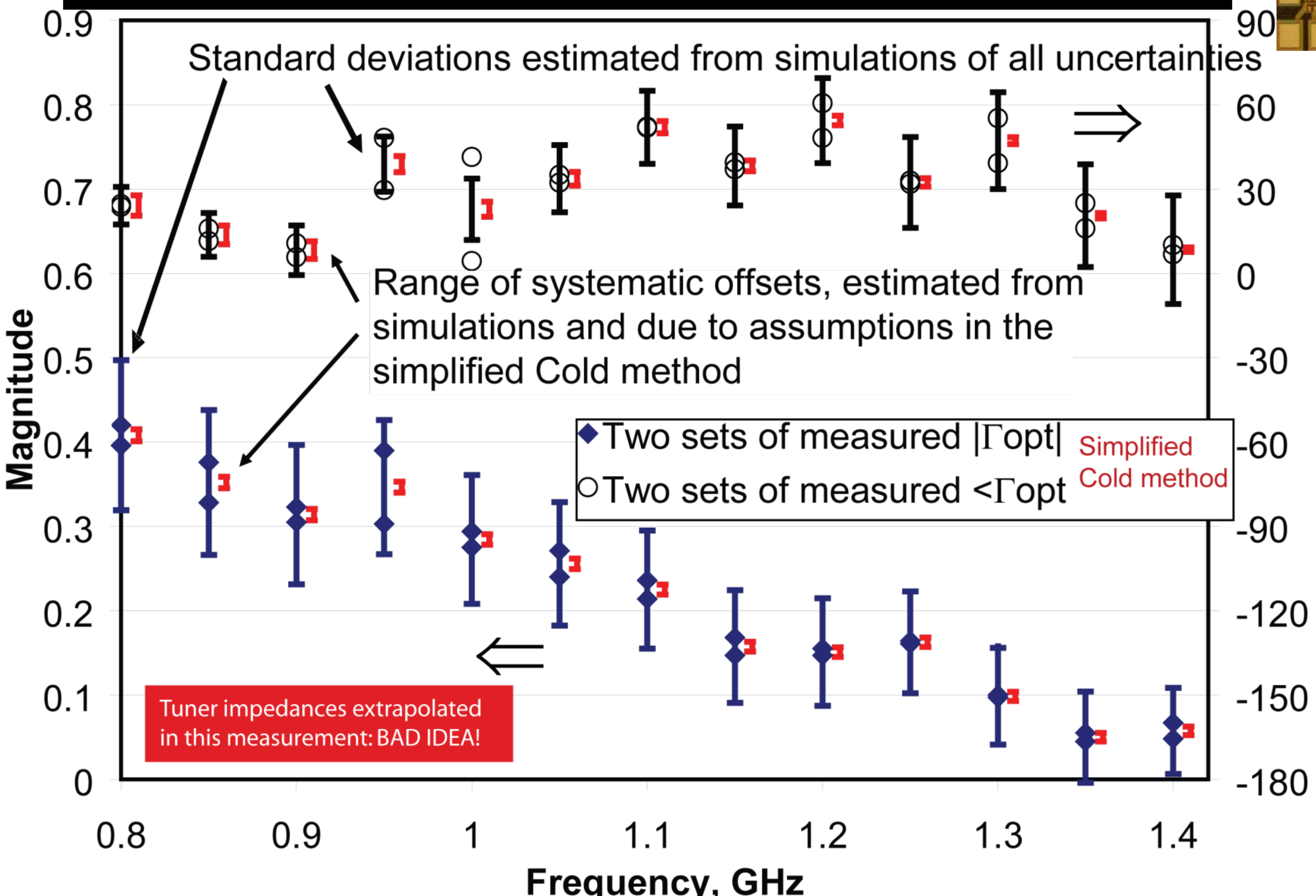


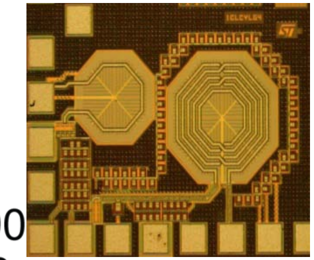
SKA LNA example





SKA LNA example





Conclusions

- A combination of noise extraction methods produces the best results in terms of smallest standard deviations
- Tuner repeatability and noise power measurements are the most critical contributors to measurement errors
- S-parameter measurements are sufficiently accurate and don't contribute significantly to noise parameter extractions
- Errors in ENRs contribute directly to systematic offsets

Thank you for your attention!