Comparison of tuner-based noise-parameter extraction methods for measurements of room-temperature SKA LNAs

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Outline

• Review: Two-port networks
• Noise-parameter measurement
• Equipment measurement accuracy
• Monte Carlo analysis
• Experimental results
• SKA LNA example
• Conclusions
Review: Two-port networks

• A single-ended low noise amplifier (LNA) can be represented by a two-port network

\[
\begin{bmatrix}
Z \\
Y \\
I \\
V \\
\end{bmatrix} = \begin{bmatrix}
V_1 \\
I_1 \\
I_2 \\
V_2 \\
\end{bmatrix}
\]

• The electrical performance of two-port is analyzed with:
  - Z-parameters \([V] = [Z][I]\)
  - Y-parameters \([I] = [Y][V]\)
  - S-parameters \([b] = [s][a]\) where \(a\) is the incident wave and \(b\) is the reflected wave
  - ABCD parameters \(\begin{bmatrix}
V_1 \\
I_1 \\
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D \\
\end{bmatrix}\begin{bmatrix}
V_2 \\
I_2 \\
\end{bmatrix}\)
  - Etc.

• These electrical parameters are represented by 2x2 matrices
Review: Two-port networks

- Similarly to electrical parameters, noise parameters of two-ports can be expressed with 2x2 matrices.
- The noisy two port networks are represented by a noiseless two-port network (discussed above) and two noise sources.

\[
\begin{align*}
C_Y &= \frac{1}{2\Delta f} \begin{vmatrix} i_1i_1^* & i_1i_2^* \\ i_1^*i_2 & i_2^*i_2 \end{vmatrix} \\
C_Z &= \frac{1}{2\Delta f} \begin{vmatrix} v_1v_1^* & v_1v_2^* \\ v_1^*v_2 & v_2^*v_2 \end{vmatrix} \\
C_{ABCD} &= \frac{1}{2\Delta f} \begin{vmatrix} v_nv_n^* & v_ni_n^* \\ v_n^*i_n & i_n^*i_n \end{vmatrix}
\end{align*}
\]

- The noise sources may be correlated.
Review: Noisy two-port networks

- Why does one need the noise correlation matrices?
  - Consider a two-port network driven by a signal source

  \[ C_{ABCD} = \frac{1}{2\Delta f} \begin{vmatrix} v_n v_n^* & v_n i_n^* \\ v_n^* i_n & i_n i_n^* \end{vmatrix} = 2kT \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} \]

  - The noise figure (factor) of this two-port is:

  \[
  F = \frac{i_s^2 + |i_n + Y_s v_n|^2}{i_s^2} = 1 + \frac{(i_n + Y_s v_n)(i_n + Y_s v_n)^*}{i_s^2} = 1 + \frac{i_n i_n^* + Y_s v_i^* i_n + Y_i i_n^* + Y_s v_i v_i^*}{i_s^2}
  \]

  and since \( i_s^2 = 4kT\Delta f \text{ Re}\{Y_s\} \) then \( F = 1 + \frac{c_{22} + Y_s c_{12} + Y_s^* c_{21} + |Y_s|^2 c_{11}}{\text{ Re}\{Y_s\}} \)

  - For a given \( Y_s \), the noise figure of a two-port network is obtained from \( C_{ABCD} \)! Nothing else is required!
Review: Two-port networks

• The noise figure (factor) of this two-port was found to be:

\[ F = 1 + \frac{c_{22} + Y_s c_{12} + Y_s^* c_{21} + |Y_s|^2 c_{11}}{\text{Re}\{Y_s\}} \]

• Can solve for \( Y_{s,\text{opt}} = G_{s,\text{opt}} + j B_{s,\text{opt}} \) that minimizes the noise figure

\[ Y_{s,\text{opt}} = \sqrt{\frac{c_{22}}{c_{11}} - \left( \frac{\text{Im}\{c_{12}\}}{c_{11}} \right)^2} + j \frac{\text{Im}\{c_{12}\}}{c_{11}} \]

• The lowest (minimum) noise figure is obtained by substituting \( Y_{s,\text{opt}} \) back into \( F \):

\[ F_{\text{min}} = 1 + \frac{c_{22} + Y_{s,\text{opt}} c_{12} + Y_{s,\text{opt}}^* c_{21} + |Y_{s,\text{opt}}|^2 c_{11}}{\text{Re}\{Y_{s,\text{opt}}\}} = 1 + 2\left( \text{Re}\{c_{12}\} + G_{s,\text{opt}} c_{11} \right) \]

• If \( Y_s \neq Y_{s,\text{opt}} \) then the noise figure (factor) is \( F = F_{\text{min}} + \frac{c_{11}}{\text{Re}\{Y_s\}} |Y_s - Y_{s,\text{opt}}|^2 \)
Review: Two-port networks

- $Y_{s,opt}$, $F_{min}$, and $F$ can be found directly by using the noise correlation matrix:

$$C_{ABCD} = 2kT \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix}$$

$$Y_{s,opt} = \sqrt{\frac{c_{22}}{c_{11}} - \left(\frac{\text{Im}\{c_{12}\}}{c_{11}}\right)^2} + j\frac{\text{Im}\{c_{12}\}}{c_{11}}$$

$$F_{min} = 1 + 2(\text{Re}\{c_{12}\} + G_{s,opt}c_{11})$$

$$F = F_{min} + \frac{c_{11}}{\text{Re}\{Y_s\}}|Y_s - Y_{s,opt}|^2$$

- Commonly $Y_{s,opt}$, $F_{min}$, and $R_n = c_{11}$ are called the noise parameters of a two-port network.

- In terms of noise parameters, the noise correlation matrix is

$$C_{ABCD} = 2kT \begin{vmatrix} R_n & F_{min} - 1 - R_nY_{s,opt}^* \\ F_{min} - 1 - R_nY_{s,opt} & R_n|Y_{s,opt}|^2 \end{vmatrix}$$
Noise-parameter measurement

- There are a few techniques: tuner based, mismatched line based, and noise wave-amplitude based, etc.
- This talk focuses on tuner-based extraction techniques.

![Diagram of noise-parameter measurement setup](image)
Noise-parameter measurement

- Tri-state noise source
- LN2 (outside temporary)
- PNA-X
- NFA
- Temperature monitor
- Shielded box
- Source tuner (Maury)
- Helium
- LNA
- Source tuner (Focus)
Noise-parameter measurement

- Tuner can generates a number of impedances (admittances) at the receiver (or the DUT) input

- Remember

\[ F = F_{\text{min}} + \frac{R_n}{G_s} \left| Y_{opt} - Y_s \right|^2 \]

- Measurement of noise figures at a few of these admittances allows the extraction of \((F_{\text{min}}, R_n \text{ and } Y_{opt})\)
Noise-parameter measurement

• How are the noise figures measured?
• Two approaches:
  – Cold methods
  – Hot-cold methods (aka modified Y-factor methods)
Noise-parameter measurement (Cold methods)

- Cold methods:
  - Noise source **only** toggles from OFF to ON when tuner synthesizes 50Ohm impedance and the receiver measures two noise power levels

\[
\begin{align*}
P^{c}_{\text{cal}} &= \left( P^{c}_{\text{in}} + \frac{T^{c}_{\text{a,0}}}{T_0} P^{c}_{\text{rec,cal}} \right) M^{c} G^{c}_{\text{rec}} \\
\frac{P^{h}_{\text{cal}}}{P^{h}_{\text{in}}} &= \left( P^{h}_{\text{in}} + \frac{T^{h}_{\text{a,0}}}{T_0} P^{h}_{\text{rec,cal}} \right) M^{h} G^{h}_{\text{rec}}
\end{align*}
\]

where \( M^{c(h)} G^{c}_{\text{rec}} \) is the receiver transducer gain, \( P^{c(h)}_{\text{in}} \) noise power at the receiver input, \( P^{c(h)}_{\text{rec,cal}} \) receiver noise power, and \( T^{c(h)}_{\text{iNet}} \) and \( G^{c(h)}_{\text{A,iNet}} \) are the noise temperature and available gain of the tuner.

- With a few different impedances at the receiver input, \( G^{c}_{\text{rec}} \) and the receiver noise parameters are obtained

- Note: noise source impedance changes when it toggles which complicates the extraction of \( G^{c}_{\text{rec}} \).
Noise-parameter measurement (Cold methods)

• Based on the way $G_{\text{rec}}$ is found, cold methods are subdivided into:
  – Simplified Cold method (assumes noise source impedance does not change when it toggles and assumes $T_a = T_{ns}$)
  – Iterative Cold method (uses iterative approach to finding $G_{\text{rec}}$)
  – Direct Cold method (uses least-squares approach to find $G_{\text{rec}}$ and the noise parameters simultaneously, similar to the modified Y-factor method)

• For DUT measurements the noise source is always OFF.
  – Noise powers measured by receiver are related to the DUT noise powers through $G_{\text{rec}}$.
  – Errors in $G_{\text{rec}}$ cause errors in noise parameters.
Noise-parameter measurement (modified Y-factor)

- Modified Y-factor method (Hot-Cold method):
  - Noise source toggles from OFF to ON at impedance synthesized by the tuner
  - The receiver measures two noise power levels for each tuner impedance

\[
\begin{align*}
P_{\text{cal}}^c &= \left( P_{\text{in}}^c + \frac{T_{c,0}^c}{T_0} P_{\text{rec,cal}}^c \right) M^c G_{\text{rec}} \\
P_{\text{cal}}^h &= \left( P_{\text{in}}^h + \frac{T_{h,0}^h}{T_0} P_{\text{rec,cal}}^h \right) M^h G_{\text{rec}}.
\end{align*}
\]

- \( G_{\text{rec}} \) and the noise parameters are determined using a least-squares fit
- For each impedance synthesized by the tuner, \( G_{\text{rec}} \) is indirectly measured \( \rightarrow \) expect better accuracy in extracting \( G_{\text{rec}} \).
# Equipment measurement accuracy

**Measurement Uncertainties Added to Evaluate Noise Parameter Extraction Methods Based on the Instrument Specifications Noted in the Reference Column**

<table>
<thead>
<tr>
<th>Measured parameter</th>
<th>Uncertainty</th>
<th>Noise model</th>
<th>Note</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-parameters</td>
<td>mean=−65 dB \ max=−60 dB</td>
<td>Rayleigh</td>
<td>High power, Notes 1,2</td>
<td>[60]</td>
</tr>
<tr>
<td></td>
<td>mean=−55 dB \ max=−50 dB</td>
<td>Rayleigh</td>
<td>Low power, Notes 1,2,3</td>
<td></td>
</tr>
<tr>
<td>Tuner repeatability</td>
<td>mean=−50 dB \ max=−44 dB</td>
<td>Rayleigh</td>
<td>Notes 4,5</td>
<td>[61]–[63]</td>
</tr>
<tr>
<td>$T_c$ measurement accuracy</td>
<td>mean=0.25 °C \ st. dev.=0.33 °C</td>
<td>Gaussian</td>
<td>Note 6</td>
<td>[64]</td>
</tr>
<tr>
<td>Receiver accuracy</td>
<td>0.017 dB</td>
<td>Gaussian</td>
<td>Note 7</td>
<td>[65]</td>
</tr>
<tr>
<td>Noise source ENR</td>
<td>0.134 dB</td>
<td>Constant offset</td>
<td>Note 8</td>
<td>[64]</td>
</tr>
</tbody>
</table>

Note 1: Since the uncertainties in the real and imaginary parts of S-parameter measurements are correlated by an unknown amount [66], the uncorrelated real and imaginary parts of S-parameters were selected as they produce larger errors in the extracted noise parameters.

Note 2: Values are based on the authors calibration experience with TRL kits using 128 averages and [60].

Note 3: When measuring the receiver input reflection coefficient and the DUT S-parameters, the VNA output power needs to be backed off in order to avoid compression of the two devices. Depending on linearity specifications of the DUT, power levels of less than −40 dBm are required.

Note 4: It was assumed that phase errors and magnitude errors are not correlated.

Note 5: Some automatic tuner vendors allow users to perform noise measurements at tuner locations that have not been calibrated but rather the supplied software estimates the actual reflection coefficient values based on measured surrounding points. Based on our measurements the estimated points worsen the tuner repeatability to a mean value of −35 dB with the maximum difference reaching −22 dB.

Note 6: To assess the effect of errors in measurements of $T_a$ and $T_{ns}$, a constant offset representing a reasonable accuracy of temperature measurements of 0.25 K was introduced for both temperature “readings” used by the Matlab program to extract the noise parameters.

Note 7: When the noise figure analyzer bandwidth is 4 MHz.

Note 8: Average ENR uncertainty for an Agilent noise source N4000A in the authors’ possession.
Monte Carlo analysis

<table>
<thead>
<tr>
<th>Simulation Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Noise Source</strong></td>
</tr>
<tr>
<td>$\Gamma_{ns}^c$</td>
</tr>
<tr>
<td>0.022°144</td>
</tr>
<tr>
<td><strong>Receiver</strong></td>
</tr>
<tr>
<td>$\Gamma_{rec}$</td>
</tr>
<tr>
<td>0.057°−136</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>SUT</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}$</td>
</tr>
<tr>
<td>0.3°50</td>
</tr>
</tbody>
</table>
Monte Carlo analysis

- Simple Cold method gives the worst performance
- Modified Y-factor is the best for Rn extractions but not \( \Gamma_{\text{opt}} \).
Monte Carlo analysis

- Modified Y-factor is the best for Fmin and Grec extractions but not Rn and Γopt.
Experimental results

**Receiver Noise Parameter Extraction Tests. During Test 4, $T_a$ was set to a constant 24 °C. The Noise Figure Analyzer Bandwidth was set to 1 MHz.**

<table>
<thead>
<tr>
<th>Test</th>
<th>Tuner calibration</th>
<th>$\Gamma_{rec}$ meas.</th>
<th>Receiver ave.</th>
<th>$T_a$ meas.</th>
<th>No of meas.</th>
<th>Constellation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Yes</td>
<td>Yes</td>
<td>32</td>
<td>Yes</td>
<td>25</td>
<td>Van den Bosch</td>
</tr>
<tr>
<td>2</td>
<td>No</td>
<td>No</td>
<td>32</td>
<td>Yes</td>
<td>25</td>
<td>Van den Bosch</td>
</tr>
<tr>
<td>3</td>
<td>No</td>
<td>No</td>
<td>128</td>
<td>Yes</td>
<td>25</td>
<td>Van den Bosch</td>
</tr>
<tr>
<td>4</td>
<td>No</td>
<td>No</td>
<td>32</td>
<td>No</td>
<td>25</td>
<td>Van den Bosch</td>
</tr>
<tr>
<td>5</td>
<td>No</td>
<td>No</td>
<td>32</td>
<td>Yes</td>
<td>25</td>
<td>all 217 tuner impedances</td>
</tr>
<tr>
<td>6</td>
<td>No</td>
<td>No</td>
<td>32</td>
<td>Yes</td>
<td>150</td>
<td>Van den Bosch</td>
</tr>
</tbody>
</table>
**Experimental results**

**Standard Deviations Obtained From Measurements During Tests 1–6. The Noise Figure Analyzer Bandwidth was Set to 1 MHz**

| Test | $T_{\text{min}}, \text{K}$ | $R_n, \Omega$ | $|\Gamma_{\text{opt}}|$ | $\angle\Gamma_{\text{opt}}$ | $kBG_{\text{rec}}$ |
|------|----------------|------------|----------------|----------------|----------------|
| 1    | 21             | 0.32       | 0.0018         | 1.8°           | 2.3e-8         |
| 2    | 20             | 0.31       | 0.0019         | 1.6°           | 1.7e-8         |
| 3    | 12             | 0.22       | 0.0017         | 1.0°           | 1.4e-8         |
| 4    | 20             | 0.31       | 0.0016         | 1.8°           | 2.0e-8         |
| 5    | 24             | 0.32       | 0.0031         | 1.3°           | 2.0e-8         |
| 6    | 18             | 0.31       | 0.0017         | 1.8°           | 1.8e-8         |

- S-parameter measurements are good enough
- NFA averaging is important!!!
- Tuner repeatability is the most dominant source of error
Experimental results (150 measurements)
Experimental results (150 measurements)

Comparison of standard deviations from the four noise parameter extraction methods normalized by the corresponding data obtained with the iterative cold method. The noise figure analyzer bandwidth was set to 1 MHz.

<table>
<thead>
<tr>
<th></th>
<th>Direct Cold</th>
<th>Modified Y-factor</th>
<th>Simplified Cold</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{min}$</td>
<td>0.9</td>
<td>0.7</td>
<td>1.0</td>
</tr>
<tr>
<td>$R_n$</td>
<td>1.0</td>
<td>3.8</td>
<td>1.0</td>
</tr>
<tr>
<td>$</td>
<td>\Gamma_{opt}</td>
<td>$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\angle \Gamma_{opt}$</td>
<td>1.0</td>
<td>5.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$kBG_{rec}$</td>
<td>0.9</td>
<td>0.7</td>
<td>1.0</td>
</tr>
</tbody>
</table>

These numbers don’t capture systematic offsets seen in the previous slide. The simplified Cold method deviates the most from the other methods.
SKA LNA example

Two sets of measured $F_{\text{min}}$ [dB] using Simplified Cold method

Range of systematic offsets, estimated from simulations and due to assumptions in the simplified Cold method (no uncertainty in ENR)

Range of systematic offsets, estimated from simulations only due to the ENR uncertainty

Standard deviations estimated from simulations of all uncertainties
SKA LNA example

Range of systematic offsets, estimated from simulations and due to assumptions in the simplified Cold method.

Tuner impedances are extrapolated in this measurement: BAD IDEA!

Standard deviations estimated from simulations of all uncertainties.
SKA LNA example

Standard deviations estimated from simulations of all uncertainties

Range of systematic offsets, estimated from simulations and due to assumptions in the simplified Cold method

Two sets of measured $|\Gamma_{\text{opt}}|$ Simplified Cold method

Two sets of measured $<\Gamma_{\text{opt}}$

Tuner impedances extrapolated in this measurement: BAD IDEA!

Frequency, GHz
Conclusions

• A combination of noise extraction methods produces the best results in terms of smallest standard deviations
• Tuner repeatability and noise power measurements are the most critical contributors to measurement errors
• S-parameter measurements are sufficiently accurate and don’t contribute significantly to noise parameter extractions
• Errors in ENRs contribute directly to systematic offsets
Thank you for your attention!