Beamformer Design Methods for Phased Array Feeds

Michael Elmer, Brian D. Jeffs, Karl F. Warnick
Brigham Young University

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Presentation outline

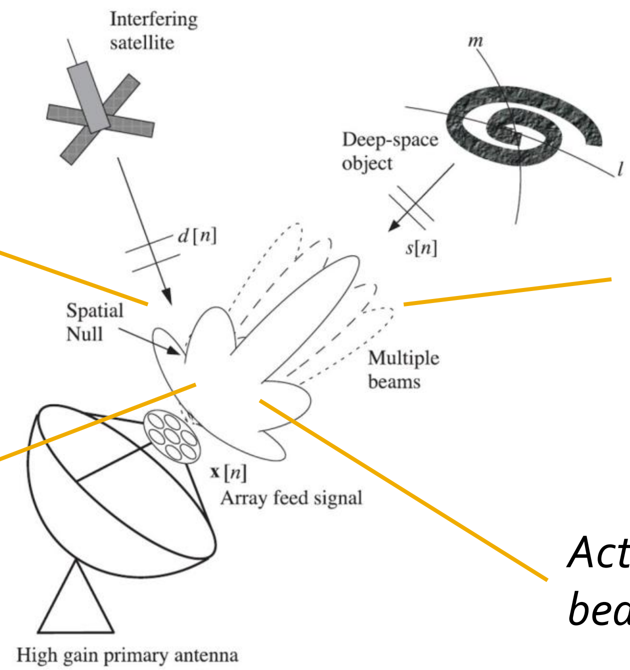
- Phased array feed (PAF) overview
  - The BYU/NRAO system
- Principles of PAF operation
- Beamformer design methods
  - Max-SNR
  - Numerically optimized
  - Hybrid
- Results
- Conclusion

PAF overview

- PAFs offer many advantages to radio astronomy.

Spatial nulling to cancel interfering signals

Adaptation to changes in the noise environment



Increased field of view

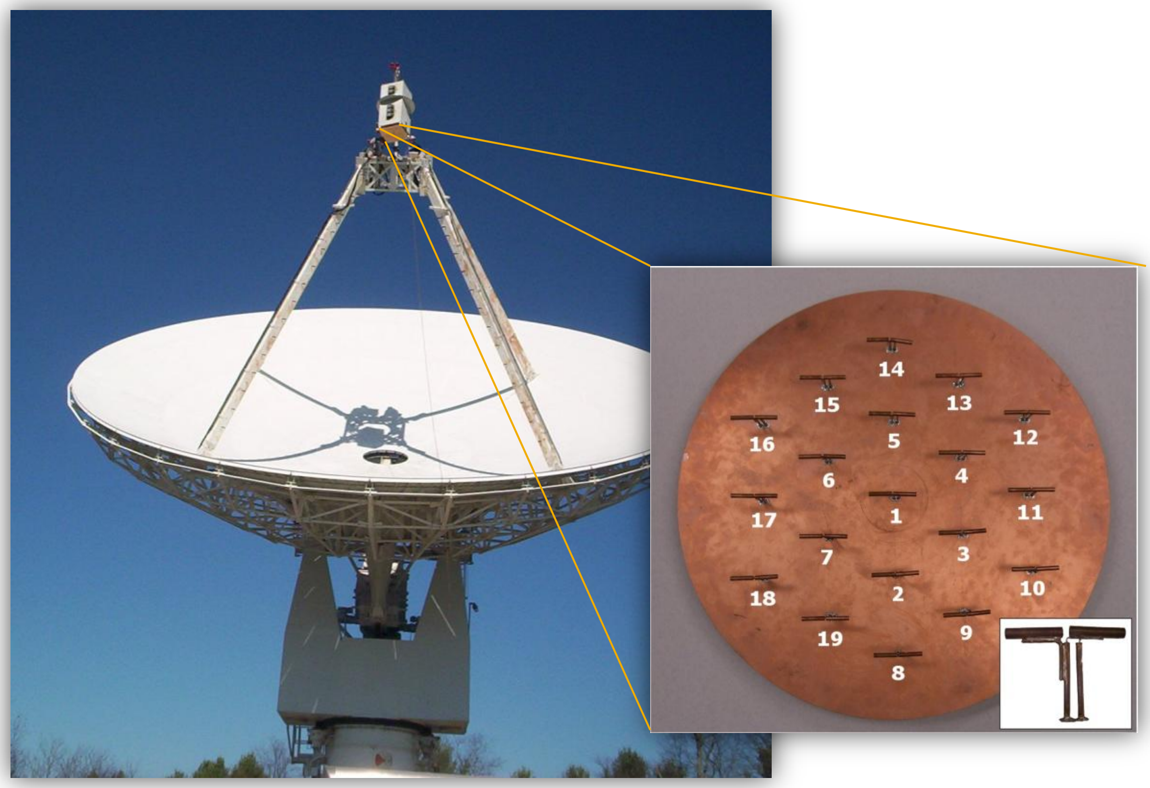
Active control of the beam pattern shape

PAF overview

- The BYU/NRAO feed on the NRAO 20 meter dish.

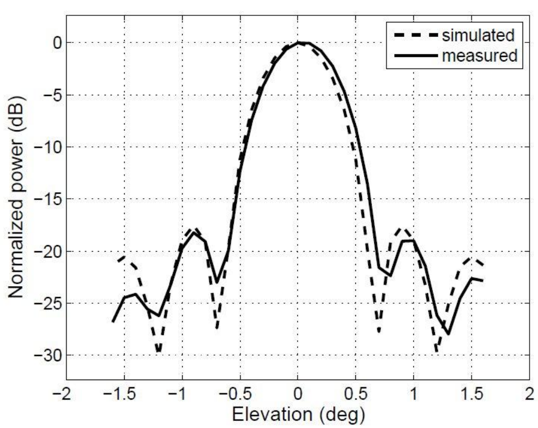
- hexagonal spacing
- 19 element half wave dipoles
- single polarization
- copper-plated ground plane
- dipoles offset by λ/4
- center frequency of 1600 MHz

- 20 meter telescope
- f/D of 0.43

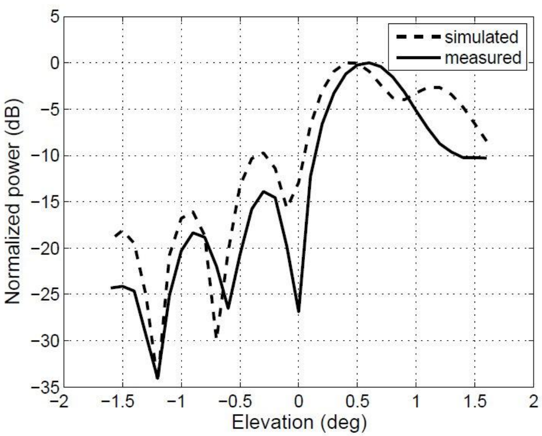


PAF overview

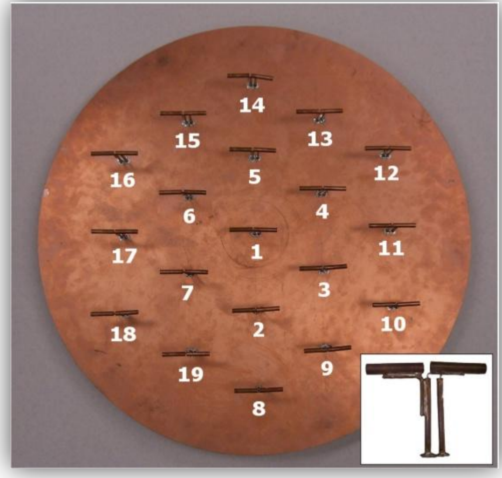
- Detailed simulation model accounts for…
  - Interaction with reflector
  - Spillover noise
  - Mutual coupling between elements



(a) Power pattern of array element 1.



(b) Power pattern of array element 16.



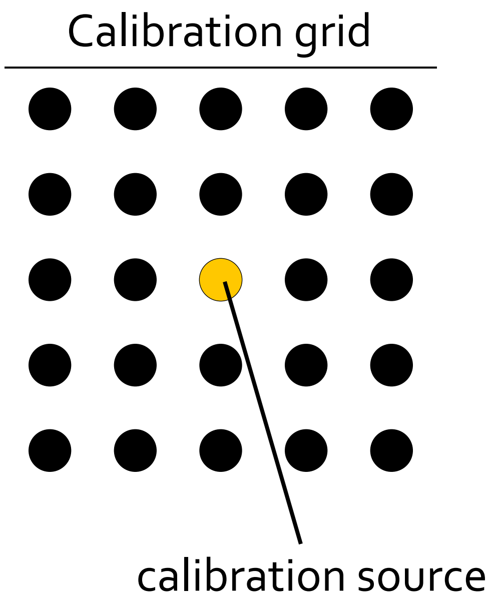
Principles of PAF operation

Calibration procedure

- A calibration vector is required for every direction in which a beam is to be steered or a response constraint is desired.
- Procedure:

1. The telescope is steered to angle θ_k relative to the calibration source.
2. A signal-plus-noise covariance $\hat{\mathbf{R}}_{\mathrm{x},k}$ is obtained.
3. The telescope is steered several degrees in azimuth and an off-source, noise only $\hat{\mathbf{R}}_{\mathrm{n},k}$ is obtained.
4. The calibration vector $\hat{\mathbf{v}}_k$ is computed as $\hat{\mathbf{v}}_k = \hat{\mathbf{R}}_{\mathrm{n}}\mathbf{u}_k$ where $\mathbf{u}_k$ is the dominant solution to

$$\hat{\mathbf{R}}_{\mathrm{x},k}\hat{\mathbf{v}}_k = \lambda_{\max}\hat{\mathbf{R}}_{\mathrm{n}}\hat{\mathbf{v}}_k$$



Principles of PAF operation

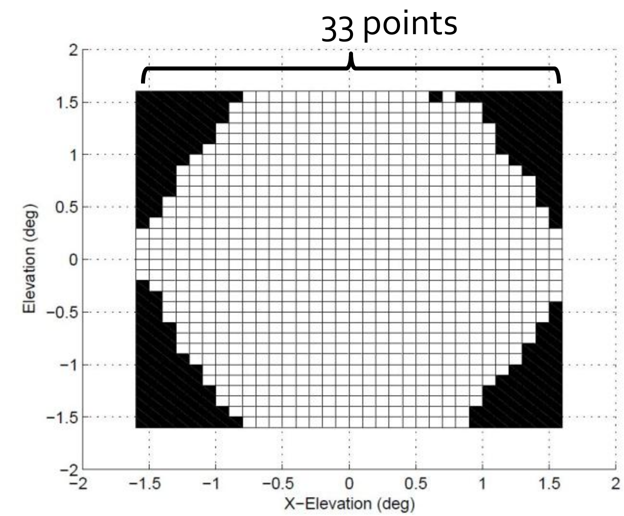
- Calibration procedure

- Quality of calibration vectors is determined by SNR and integration time.
- We use an information theoretic based algorithm to determine the quality of a calibrator, estimating the rank *d* of $\hat{\mathbf{R}}_s$ as

$$\hat{d}_{MDL} = \arg\min_{d} \left\{ L_d(d) + \frac{1}{2}[d(2P-d)+1]\ln L \right\}$$

Minimum Description Length

$$L_d(d) = L(P-d)\ln \left\{ \frac{\frac{1}{P-d}\sum_{i=d+1}^{P}\hat{\lambda}_i}{\left(\prod_{i=d+1}^{P}\hat{\lambda}_i\right)^{\frac{1}{P-d}}} \right\}$$



Beamformer design methods

- Max-SNR beamformer

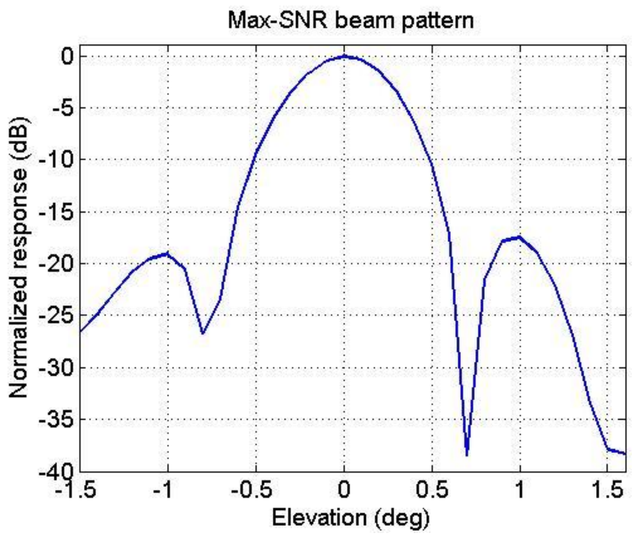
- The max-SNR beamformer is defined as

$$\mathbf{w}_{\mathrm{m}} = \arg\max_{\mathbf{w}} \frac{\mathbf{w}^H \mathbf{R}_{\mathrm{s}} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{\mathrm{n}} \mathbf{w}} \qquad (1)$$

- The maximization gives the generalized eigenvalue problem

$$\hat{\mathbf{R}}_{\mathrm{s}} \mathbf{w} = \lambda_{\mathrm{max}} \hat{\mathbf{R}}_{\mathrm{n}} \mathbf{w} \qquad (2)$$

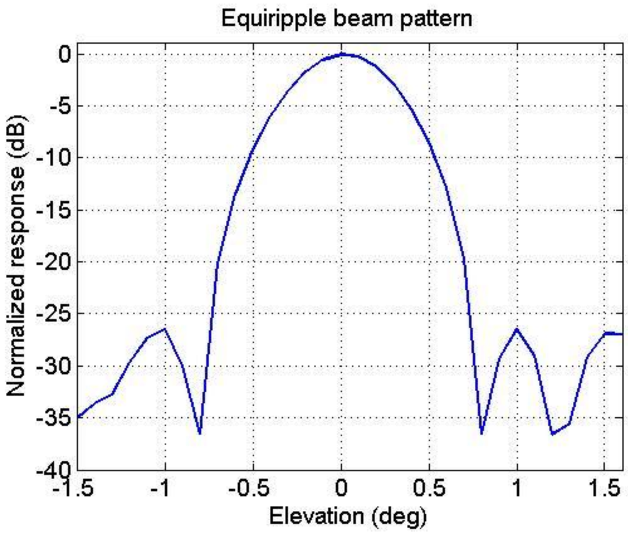
- Easy to implement.



Beamformer design methods

▪ Numerically optimized equiripple beamformer

- To provide full control of the far field beam pattern shape, we use a numerical optimizer.
  - Matlab's fmincon optimization routine was used to give minimax result.
  - Constraints within the main beam including the peak, and on the phase of the 1st beamformer element.
  - Minimization of the maximum sidelobe level is achieved by solving the objective function



$$\mathbf{w}_{\mathrm{e}} = \arg\min_{\mathbf{w}} F(\mathbf{w}) \text{ subject to}$$

$$|\mathbf{v}^H(\theta_k)\mathbf{w}| = c_k \quad \{\forall\, k, 1 \le k \le J\} \text{ and } \angle[\mathbf{w}]_1 = 0$$

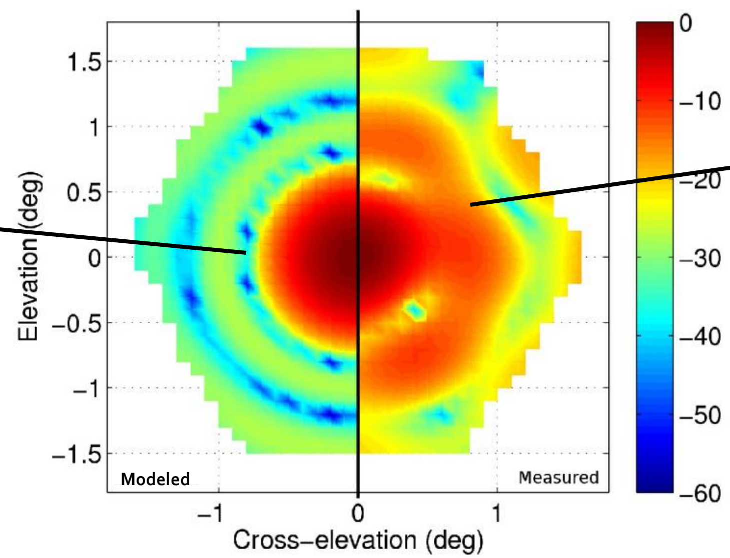
$$F(\mathbf{w}) = \max_i \left| [\mathbf{V}_{\mathrm{side}}^H \mathbf{w}]_i \right|$$

Beamformer design methods

▪ Numerically optimized equiripple beamformer

- Simulated beamformer design: done quickly with arbitrarily large and dense grid of points.
- Unfortunately, distortions in measured beam pattern.
- A transformation step is required to reduce these undesirable effects.

Modeled beamformer applied to ***modeled*** calibrators



Modeled beamformer applied to ***measured*** calibrators

Beamformer design methods

▪ Numerically optimized equiripple beamformer

- The goal is to transform the simulated beamformer to give a measured beam pattern closely matching the simulated designed beam pattern.

$$\hat{\mathbf{w}}' = \mathbf{T}\tilde{\mathbf{w}} \tag{3}$$

$$\begin{aligned} \hat{\mathbf{w}}' &= \arg\min_{\hat{\mathbf{w}}} ||\hat{\mathbf{w}}^H\hat{\mathbf{V}} - \tilde{\mathbf{w}}^H\tilde{\mathbf{V}}||_2^2 \\ &= (\hat{\mathbf{V}}^H)^\dagger\tilde{\mathbf{V}}^H\tilde{\mathbf{w}} \end{aligned} \tag{4}$$

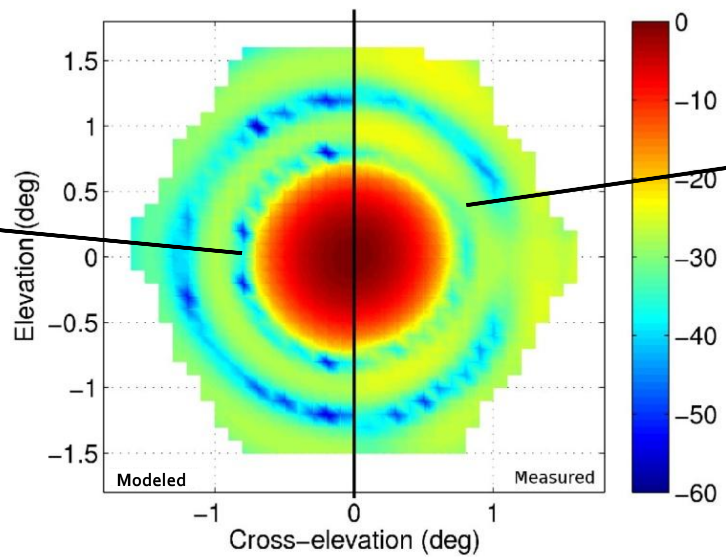
$$\mathbf{T} = [(\hat{\mathbf{V}}\mathbf{G}\hat{\mathbf{V}}^H)^{-1}\hat{\mathbf{V}}\mathbf{G}\tilde{\mathbf{V}}^H] \tag{5}$$

Beamformer design methods

▪ Numerically optimized equiripple beamformer

- The transformed pattern matches simulated pattern.
- A full grid of measured calibrators is required.
- Deterministic PAF beamformers are best done with measured calibrators.

Modeled beamformer applied to ***modeled*** calibrators



Transformed beamformer applied to ***measured*** calibrators

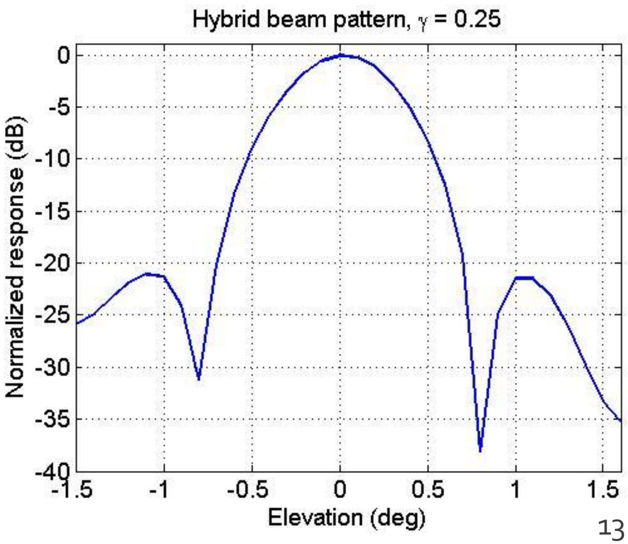
Beamformer design methods

▪ Hybrid beamformer

- A hybrid beamformer offers the best sensitivity for a given amount of beam pattern control.
- The beamformer is obtained using the numerical optimizer to solve

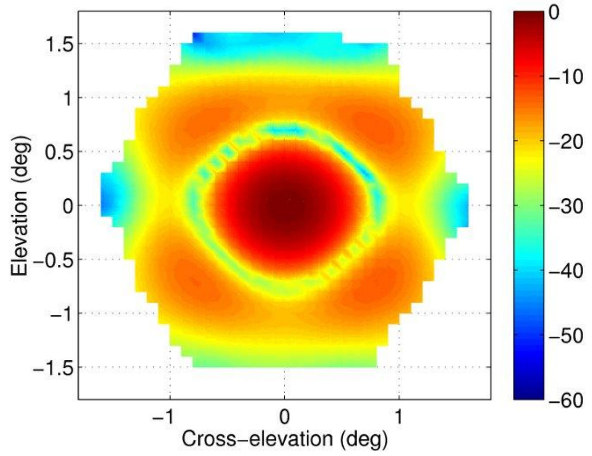
$$\mathbf{w}_\mathrm{h} = \arg\min_{\mathbf{w}} U(\mathbf{w}) \text{ subject to}$$
$$\left| |\mathbf{v}^H(\theta_k)\mathbf{w}| - c_k \right| \le 2\gamma |a_k - c_k| \;\; \{\forall k, 1 \le k \le J\}$$
$$\text{and } \angle[\mathbf{w}]_1 = 0$$

$$U(\mathbf{w}) = \gamma \frac{1}{\mathrm{SNR}(\mathbf{w})} + (1-\gamma) \max_i \left| [\mathbf{V}_\mathrm{side}^H \mathbf{w}] \right|_i$$



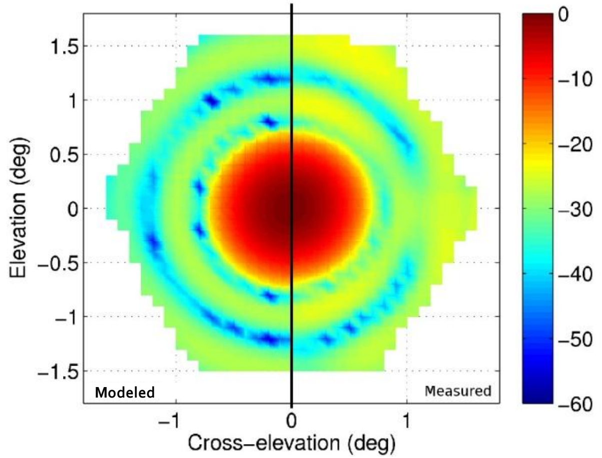
Results

▪ Comparison of beamformer methods



<u>Max-SNR beamformer</u>

- high, 13 dB side lobes
- pattern is non-uniform, uncontrolled

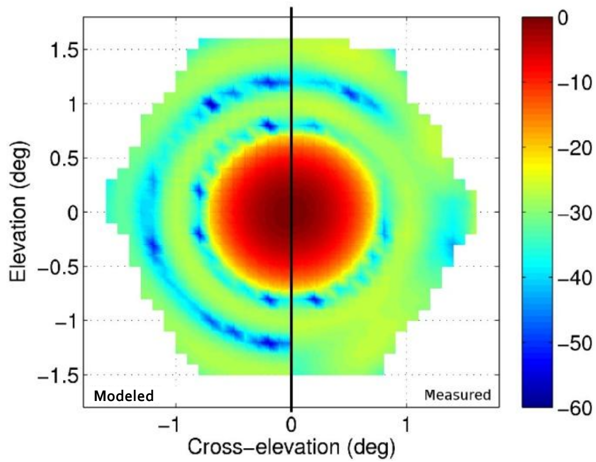


<u>Transformed equiripple beamformer</u>

- low, 26 dB side lobes
- uniform pattern
- slight distortions are visible

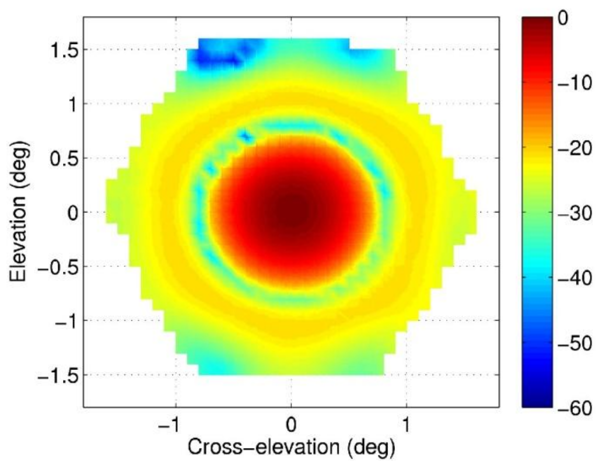
Results

▪ Comparison of beamformer methods



Measured equiripple beamformer

- low, 26 dB side lobes
- uniform pattern
- less distortion than transformed pattern



Hybrid beamformer

- γ = 0.25
- 20 dB side lobes
- pattern is uniform

Results

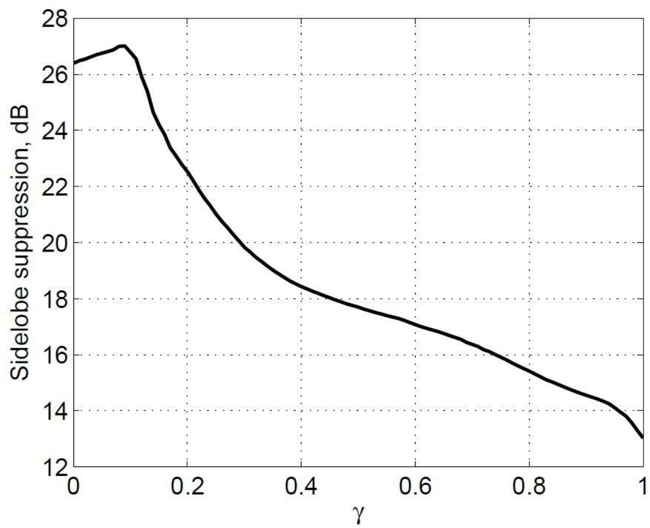
- Comparison of beamformer methods

Beamformer	Beamwidth	Peak side lobes	Sensitivity
max-SNR	$1.6°$	-13.03 dB	$2.973\ \mathrm{m}^2/K$
equiripple	$1.6°$	-26.40 dB	$1.839\ \mathrm{m}^2/K$
hybrid ($\gamma = 0.5$)	$1.6°$	-17.70 dB	$2.860\ \mathrm{m}^2/K$
hybrid ($\gamma = 0.25$)	$1.6°$	-21.02 dB	$2.545\ \mathrm{m}^2/K$

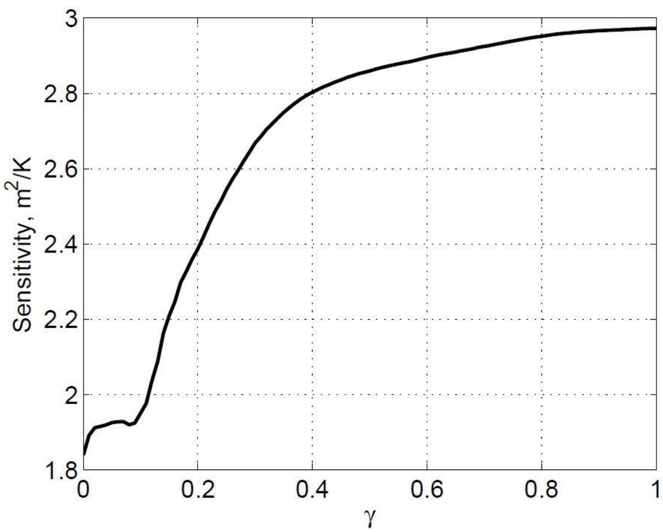
Results

▪ Effects of varying γ value

What are the effects of varying the parameter γ?



Sidelobe level, dB



Sensitivity, m^2/K

Results

▪ Value of simulated beamformers

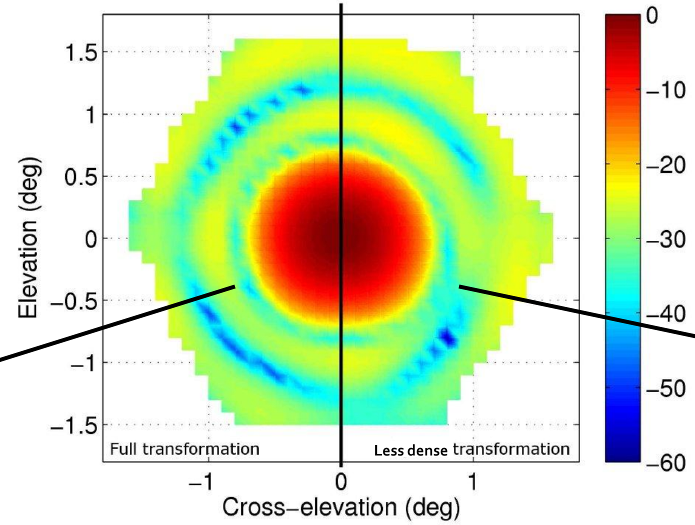
- Measured calibrators are needed for a transformation of simulated beamformer weights, but the measured calibrators can be used directly to design the beamformer.
- Simulated beamformer design may still offer an advantage when…
  - Optimizing over a more dense grid of calibrators than is needed for a transformation.
  - Having simulated calibration points on an increased span of angles.

Results

▪ Value of simulated beamformers

- The highly oversampled 33x 33 grid we obtained contains 817 good points.
- A reduction to 86 points represents a savings of 3 hours for calibration.
- Same result with fewer transformation points (good!), however…

Pattern designed using full simulation grid, then transformed using full measured grid.
Sensitivity: 1.738 m²/K.



Pattern designed using full simulation grid, then transformed using less dense measured grid.
Sensitivity: 1.712 m²/K.

Results

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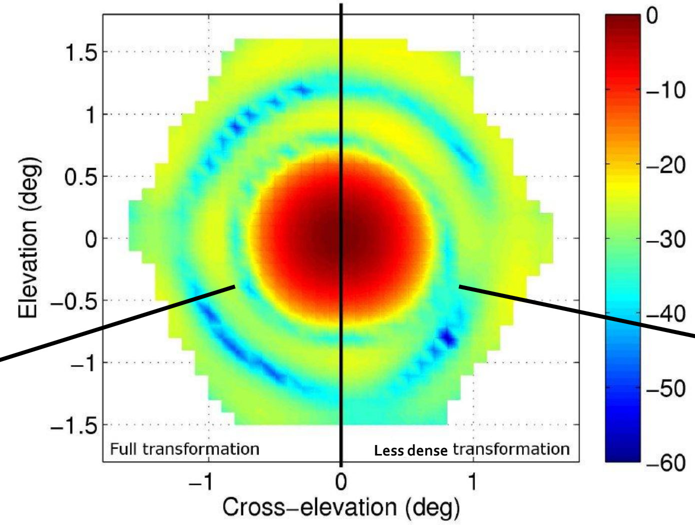


Pattern designed using full simulation grid, then transformed using less dense measured grid.
Sensitivity: 1.712 m²/K.

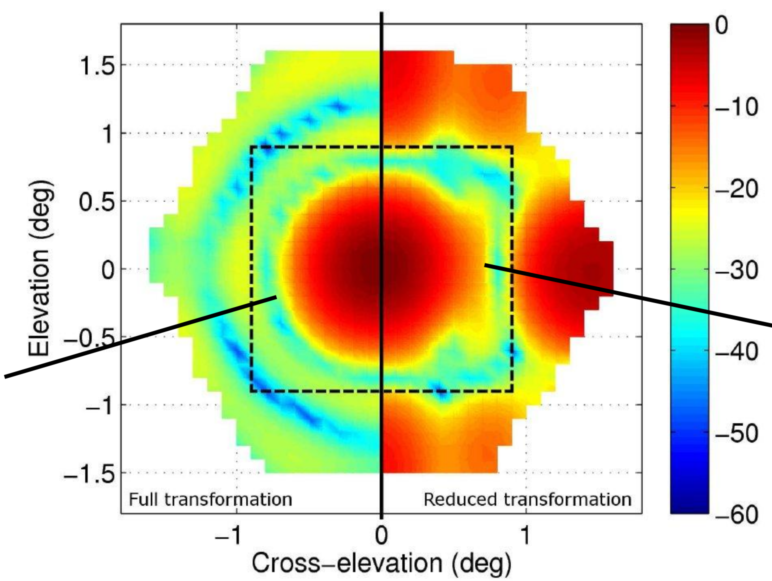
--- When using sparse grid of measured calibrators directly the sensitivity is 1.801 m²/K. ---

Results

▪ Value of simulated beamformers

- Designing with a larger span of grid points would be useful if we can retain the beam shape control in the extended region.
- Control is forfeited outside the region of the transformation set.

Pattern designed using full simulation grid, then transformed using full measured grid.



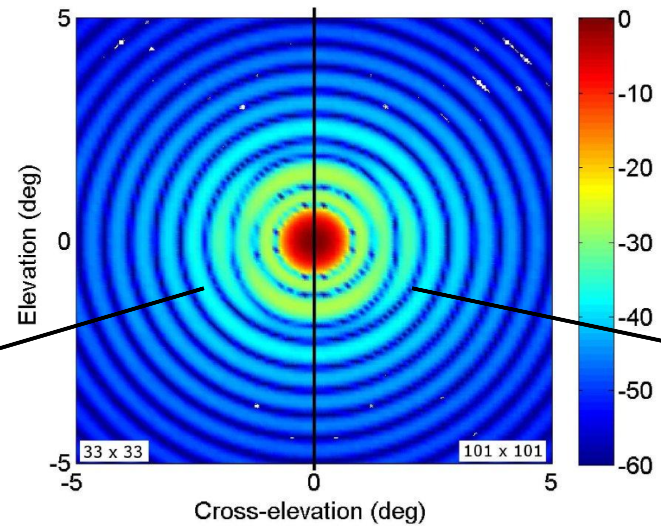
Pattern designed using full simulation grid, then transformed using reduced measured grid.

Results

▪ Angular limits of pattern control

- There does not exist an unreachable region of the beam pattern between the reach of the calibration set and the beginning of the dish aperture pattern dominance.

Beamformer designed using 33x33 grid, then applied to a 101x101 grid.



Beamformer designed using a 101x101 grid, then applied to a 101x101 grid.

Conclusion

- Simulated PAF beamformer design is possible but requires a transformation step.
- An algorithm for determining the quality of a PAF calibration vector was introduced.
- Both suitable sensitivity and beam pattern shape control can be achieved using the proposed hybrid beamforming method.
- There is no advantage to simulated beamformer design because the measured pattern is so highly dependent on the required transformation.
- Measured calibration vectors can be obtained across the limited field of view imposed by the reflector dish.